

Social Common Capital and Sustainable Development

H. Uzawa

Social Common Capital Research, Tokyo, Japan

(IPD Climate Change Manchester Meeting)

In this paper, we prove in terms of the prototype model of social common capital that the optimum conditions for sustainable processes of capital accumulation involving both private capital and social common capital coincide precisely with those for market equilibrium with the social common capital taxes at certain specific rates under the stationary expectations hypothesis concerning the future schedule of marginal productivity of capital of all kinds.

Social common capital involves intergenerational equity and justice. Although the construction and maintenance of social common capital require the use of substantial portions of scarce resources, both human and non-human, putting a significant burden on the current generation, but the people in future generations will benefit greatly if the construction of social common capital carried out by the current generation is properly arranged.

We discuss the problems of the accumulation of social common capital primarily

from the viewpoint of the intergenerational distribution of utility. Our analysis is based on the concept of sustainability introduced in Uzawa (2003, 2005), and we examine the conditions under which processes of the accumulation of social common capital over time are sustainable. The conceptual framework of the economic analysis of social common capital developed in Uzawa (2005) is extended to deal with the problems of the irreversibility of processes of the accumulation of social common capital due to the Penrose effect.

We formulate the concept of sustainability within the theoretical framework of the economic analysis of social common capital in such a manner that it may be consulted in devising institutional arrangements and policy measures in realizing the stationary state in the sense introduced by John Stuart Mill in his classic *Principles of Political Economy* (Mill, 1848), particularly in the chapter entitled "Of the Stationary State." The stationary state, as envisioned by Mill, is interpreted as the state of the economy in which all macro-economic variables, such as gross domestic product, national income, consumption, investments, prices, wages, and real rates of interest, remain at the sustainable levels, whereas, within the society, individuals are actively engaged in economic, social, and cultural activities, new scientific discoveries are incessantly made, and new products are continuously introduced while the natural environment is being preserved at the sustainable state.

Imputed Price of Capital and Sustainability

The imputed price ψ_t of each kind of capital at a particular time t expresses the extent to which the marginal increase in the stock of that kind of capital at time t induces the marginal increase in units of market prices in the welfare level of the country concerned, including those of all future generations. The imputed price ψ_t is defined to be at the sustainable level at time t , if it remains stationary at time t ; i. e.,

$$\dot{\psi}_t = 0 \text{ at time } t,$$

where $\dot{\psi}_t$ refers to the time derivative with respect to the time of the virtual capital market at time t . A time-path of the accumulations of capital is defined *sustainable*, if the imputed price π_t is at the sustainable level at all times t .

Because the sustainability of the imputed price is defined with respect to the fictitious time of the virtual capital market at time t , the sustainability of the imputed price does not necessarily imply the stationarity of the imputed price, as inadvertently stated in Uzawa (2003, 2005). All the policy and institutional conclusions obtained there, however, remain valid with regard to the concept of the sustainability of time-paths of the processes of capital accumulation, both private capital and social common capital.

The imputed price of capital at time t , ψ_t , is the discounted present value of the

marginal increases in outputs in the future measured in units of the utility due to the marginal increase in the stock of capital at time t . The marginal increase in outputs at future time τ in units of the utility is given by $m_\tau \psi_\tau$, where m_τ is the marginal efficiency of investment at future time τ .

If we denote the social rate of discount and the rate of depreciation of capital, respectively, by δ and μ , the imputed price of capital at time t , ψ_t , is given by

$$\psi_t = \int_t^\infty m_\tau \psi_\tau e^{-(\delta + \mu)(\tau - t)} d\tau,$$

which, by differentiating with respect to time t , we obtain the following differential equation:

$$\dot{\psi}_t = (\delta + \mu)\psi_t - m_t \psi_t. \quad (1)$$

Differential equation (1) is nothing but the Euler-Lagrange differential equation in the calculus of variations. In the context of the theory of optimum capital accumulation, it is often referred to as the Ramsey-Keynes equation. The economic meaning of the Ramsey–Keynes equation (1) may be brought out better if we rewrite it as

$$\dot{\psi}_t + m_t \psi_t = (\delta + \mu)\psi_t. \quad (2)$$

We suppose that capital is transacted as an asset on a virtual capital market that is perfectly competitive and the imputed price ψ_t is identified with the market price at time t .

Consider the situation in which the unit of such an asset is held for the short time period

$[t, t + \Delta t]$ ($\Delta t > 0$). The gains obtained by holding such an asset are composed of "capital gains" $\Delta\psi_t = \psi_{t+\Delta t} - \psi_t$ and "earnings" $m_t\psi_t\Delta t$; that is,

$$\Delta\psi_t + m_t\psi_t\Delta t.$$

On other hand, the cost of holding such an asset for the time period $[t, t + \Delta t]$ consists of "interest payments" $\delta\psi_t\Delta t$ and "depreciation charges" $\mu\psi_t\Delta t$, where the social rate of discount δ is identified with the market rate of interest. Hence, on the virtual capital market, these two amounts become equal; that is,

$$\Delta\psi_t + m_t\psi_t\Delta t = (\delta + \mu)\psi_t\Delta t.$$

By dividing both sides of this equation by Δt and taking the limit as $\Delta t \rightarrow 0$, we obtain relation (2).

The imputed price ψ_t is defined to be at the sustainable level at time t , if it remains stationary at time t ; i. e.,

$$\dot{\psi}_t = 0 \text{ at time } t,$$

where $\dot{\psi}_t$ refers to the time derivative with respect to the fictitious time of the virtual capital market at time t . Hence, the imputed price π_t is at the sustainable level at time t , if, and only if, the marginal efficiency of investment is equal to the sum of the social rate of discount and the rate of depreciation; i. e.,

$$m_t = \delta + \mu \text{ at time } t.$$

A time-path of capital accumulation is defined *sustainable*, if it is at the sustainable level at all times t .

The Prototype Model of Social Common Capital

In the prototype model of social common capital introduced in Uzawa (2005), we consider a particular type of social common capital – social infrastructure, such as public utilities, public transportation systems, ports, and highways. We consider the general circumstances where factors of production that are necessary for the professional provision of services of social common capital are either privately owned or managed as if private owned. Services of social common capital are subject to the phenomenon of congestion, resulting in the divergence between private and social costs. Therefore, to obtain efficient and equitable allocation of scarce resources, it becomes necessary to levy taxes on the use of services of social common capital. The prices charged for the use of services of social common capital exceed, by the tax rates, the prices paid to social institutions in charge of the provision of services of social common capital. One of crucial problems in the economic analysis of social common capital is to examine how the optimum tax rates for the services of various components of social common capital are determined.

Basic Premises of the Prototype Model of Social Common Capital

We consider an economy consisting of n individuals, m private firms, and s social institutions in charge of social common capital. Individuals are generically denoted by $\nu = 1, \dots, n$, private firms by $\rho = 1, \dots, m$, and social institutions by $\sigma = 1, \dots, s$. Goods produced by private firms are generically denoted by $j = 1, \dots, J$. Fixed factors of production are generically denoted by $f = 1, \dots, F$, whereas there is only one kind of social common capital.

The utility of each individual ν is cardinal and is expressed by the utility function

$$u^\nu = u^\nu(c^\nu, \phi^\nu(a)a^\nu),$$

where c^ν is the vector of goods consumed and a^ν is the amount of services of social common capital used, both by individual ν , whereas a is the total amount of services of social common capital used by all members of the society:

$$a = \sum_{\nu} a^\nu + \sum_{\rho} a^\rho,$$

where a^ρ is the amount of services of social common capital used by private firm ρ . The impact index function $\phi^\nu(a)$ expresses the extent to which the utility of individual ν is affected by the phenomenon of congestion with respect to the use of services of social common capital. The impact coefficients $\tau^\nu(a)$ of social common capital defined by

$$\tau^v(a) = -\frac{\phi^v'(a)}{\phi^v(a)} > 0$$

are assumed to be identical for all individuals:

$$\tau^v(a) = \tau(a) \quad \text{for all } v.$$

The following conditions are satisfied:

$$\tau(a) > 0, \quad \tau'(a) > 0 \quad \text{for all } a > 0.$$

The utility function $u^v(c^v, a^v)$ is assumed to satisfy the following conditions:

(U1) $u^v(c^v, a^v)$ is defined, positive, continuous, and continuously twice-differentiable

with respect to (c^v, a^v) for all $(c^v, a^v) \geq 0$.

(U2) $u_{c^v}^v(c^v, a^v) > 0$, $u_{a^v}^v(c^v, a^v) > 0$ for all $(c^v, a^v) \geq 0$.

(U3) Marginal rates of substitution between any pair of consumption goods and services of social common capital are diminishing, or more specifically, $u^v(c^v, a^v)$ is strictly quasi-concave with respect to (c^v, a^v) .

(U4) $u^v(c^v, a^v)$ is homogeneous of order 1 with respect to (c^v, a^v) .

Private Firms

Processes of production of private firms are also affected by the phenomenon of congestion regarding the use of services of social common capital. We assume that, in each private firm

ρ , the minimum quantities of factors of production that are required to produce goods by x^ρ and at the same time to increase the stock of fixed factors of production by $z^\rho = (z_f^\rho)$ with the use of services of social common capital at the level a^ρ are specified by the following vector-valued function:

$$f^\rho(x^\rho, z^\rho, \varphi^\rho(a)a^\rho) = (f_f^\rho(x^\rho, z^\rho, \varphi^\rho(a)a^\rho)),$$

where $\varphi^\rho(a)$ is the impact index with regard to the extent to which the effectiveness of services of social common capital in processes of production in private firm ρ is impaired by congestion. For private firm ρ , the impact coefficients $\tau^\rho(a)$ of social common capital defined by

$$\tau^\rho(a) = -\frac{\varphi^{\rho\prime}(a)}{\varphi^\rho(a)}$$

are assumed to be identical for all private firms, identical to those for individuals:

$$\tau^\rho(a) = \tau(a) \quad \text{for all } \rho.$$

The production possibility set of each private firm ρ , T^ρ , is composed of all combinations (x^ρ, z^ρ, a^ρ) of vectors of production x^ρ and investment z^ρ , and use of services of social common capital a^ρ that are possible with the organizational arrangements, technological conditions, and given endowments of factors of production K^ρ in firm ρ . It may be expressed as

$$T^p = \{(x^p, z^p, a^p) : (x^p, z^p, a^p) \geq 0, f^p(x^p, z^p, \varphi^p(a)a^p) \leq K^p\},$$

where the total amount of services of social common capital used by all members of the society, a , is assumed to be a given parameter.

The following neoclassical conditions are assumed:

(T^p1) $f^p(x^p, z^p, a^p)$ are defined, positive, continuous, and continuously twice-differentiable with respect to (x^p, z^p, a^p) .

(T^p2) $f_{x^p}^p(x^p, z^p, a^p) > 0$, $f_{z^p}^p(x^p, z^p, a^p) > 0$, $f_{a^p}^p(x^p, z^p, a^p) < 0$.

(T^p3) $f^p(x^p, z^p, a^p)$ are strictly quasi-convex with respect to (x^p, z^p, a^p) .

(T^p4) $f^p(x^p, z^p, a^p)$ are homogeneous of order 1 with respect to (x^p, z^p, a^p) .

Social Institutions in Charge of Social Common Capital

In each social institution σ , the minimum quantities of factors of production required to provide services of social common capital by a^σ and at the same time to engage in

investment activities to accumulate the stock of fixed factors of production by $z^\sigma = (z_f^\sigma)$

with the use of produced goods by $c^\sigma = (c_j^\sigma)$ are specified by a vector-valued function:

$$f^\sigma(a^\sigma, z^\sigma, c^\sigma) = (f_f^\sigma(a^\sigma, z^\sigma, c^\sigma)).$$

For each social institution σ , the production possibility set T^σ is composed of all

combinations $(a^\sigma, z^\sigma, c^\sigma)$ of provision of services of social common capital a^σ , investment z^σ , and use of produced goods c^σ that are possible with the organizational arrangements, technological conditions, and the given endowments of factors of production in social institution σ , K^σ . That is, it may be expressed as

$$T^\sigma = \{(a^\sigma, z^\sigma, c^\sigma) : (a^\sigma, z^\sigma, c^\sigma) \geq 0, f^\sigma(a^\sigma, z^\sigma, c^\sigma) \leq K^\sigma\}.$$

The following neoclassical conditions are assumed:

(T^σ1) $f^\sigma(a^\sigma, z^\sigma, c^\sigma)$ are defined, non-negative, continuous, and continuously twice-differentiable with respect to $(a^\sigma, z^\sigma, c^\sigma)$ for all $(a^\sigma, z^\sigma, c^\sigma) \geq 0$.

(T^σ2) $f_{a^\sigma}^\sigma(a^\sigma, z^\sigma, c^\sigma) > 0$, $f_{z^\sigma}^\sigma(a^\sigma, z^\sigma, c^\sigma) > 0$, $f_{c^\sigma}^\sigma(a^\sigma, z^\sigma, c^\sigma) < 0$ for all $(a^\sigma, z^\sigma, c^\sigma) \geq 0$.

(T^σ3) $f^\sigma(a^\sigma, z^\sigma, c^\sigma)$ are strictly quasi-convex with respect to $(a^\sigma, z^\sigma, c^\sigma)$ for all $(a^\sigma, z^\sigma, c^\sigma) \geq 0$.

(T^σ4) $f^\sigma(a^\sigma, z^\sigma, c^\sigma)$ are homogeneous of order 1 with respect to $(a^\sigma, z^\sigma, c^\sigma)$.

Capital Accumulation in the Prototype Model of Social Common Capital

The accumulation of the stock of capital goods in private firm ρ is given by the following differential equation

$$\dot{K}_t^\rho = z_t^\rho - \mu K_t^\rho, \quad K_0^\rho = K_o^\rho, \quad (3)$$

where z_t^ρ is the vector specifying the levels of investment in capital goods in private firm ρ at time t and μ is the rate of depreciation.

Similarly, the accumulation of the stock of capital goods in social institution σ is given by the following differential equation

$$\dot{K}_t^\sigma = z_t^\sigma - \mu K_t^\sigma, K_0^\sigma = K_o^\sigma, \quad (4)$$

where z_t^σ is the vector specifying the levels of investment in capital goods in social institution σ at time t and μ is the rate of depreciation.

Imputed Prices and Sustainable Processes of Capital Accumulation in the Prototype Model of Social Common Capital

Exactly as in the aggregative model of capital accumulation, the imputed price of capital in the prototype model of social common capital is defined. The imputed price, in units of the utility, of each kind of capital at time t , ψ_t , is the discounted present value of the marginal increases in total utility in the future due to the marginal increase in the stock of capital of that kind at time t . When we denote by r_τ the marginal increase in the total utility at future time τ , the imputed price at time t , ψ_t , is given by

$$\psi_t = \int_t^\infty r_\tau e^{-(\delta + \mu)(\tau - t)} d\tau. \quad (5)$$

By differentiating both sides of (5) with respect to time t , we obtain the following differential equation:

$$\dot{\psi}_t = (\delta + \mu)\psi_t - r_t. \quad (6)$$

As in the case of the aggregative model of capital accumulation, we suppose that capital is transacted as an asset on a virtual capital market that is perfectly competitive and the imputed price ψ_t is identified with the market price at time t . Consider the situation in which the unit of such an asset is held for the short time period $[t, t + \Delta t]$ ($\Delta t > 0$). The gains obtained by holding such an asset are composed of "capital gains" $\Delta\psi_t = \psi_{t+\Delta t} - \psi_t$ and "earnings" $r_t \Delta t$; that is,

$$\Delta\psi_t + r_t \Delta t.$$

On other hand, the costs of holding such an asset for the time period $[t, t + \Delta t]$ consist of "interest payments" $\delta\psi_t \Delta t$ and "depreciation charges" $\mu\psi_t \Delta t$, where the social rate of discount δ is identified with the market rate of interest; that is,

$$\delta\psi_t \Delta t + \mu\psi_t \Delta t.$$

On the virtual capital market, these two amounts become equal; that is,

$$\Delta\psi_t + r_t \Delta t = \delta\psi_t \Delta t + \mu\psi_t \Delta t.$$

By dividing both sides of this equation by Δt and taking the limit as $\Delta t \rightarrow 0$, we obtain

relation (6).

We define that the imputed price ψ_t is at the sustainable level at time t , if it remains stationary at time t ; i. e.,

$$\dot{\psi}_t = 0 \text{ at time } t,$$

where it may be reminded that $\dot{\psi}_t$ refers to the time derivative with respect to the fictitious time of the virtual capital market at time t .

From the basic differential equation (6), the imputed price ψ_t is at the sustainable level at time t , if, and only if,

$$\psi_t = \frac{r_t}{\delta + \mu} \text{ at time } t,$$

where r_t is the marginal increase in total utility due to the marginal increase in the stock of capital of that kind at time t .

With respect to the prototype model of social common capital, the imputed price of capital in private firm ρ at time t , ψ_t^ρ , is at the sustainable level at time t , if, and only if,

$$\psi_t^\rho = \frac{r_t^\rho}{\delta + \mu} \text{ at time } t, \quad (7)$$

where r_t^ρ is the marginal increase in total utility due to the marginal increase in the stock of capital in private firm ρ at time t .

Similarly, the imputed price of capital in social institution σ at time t , ψ_t^σ , is at the sustainable level at time t , if, and only if,

$$\psi_t^\sigma = \frac{r_t^\sigma}{\delta + \mu} \text{ at time } t, \quad (8)$$

where r_t^σ is the marginal increase in total utility due to the marginal increase in the stock of capital in social institution σ at time t .

A time-path of capital accumulation is defined *sustainable*, if the imputed prices of all kind of capital, private capital and social common capital, are at the sustainable levels at all times, i. e., (7) and (8) hold at all times t , for all firms ρ and social institutions σ .

Sustainable Processes of Consumption and Investment

We presume that the imputed prices of capital goods in private firms and social institutions in charge of social common capital, all at time t , are given, respectively, by ψ_t^ρ and ψ_t^σ .

Then the imputed real national income in units of the utility at time t is given by

$$H_t = \sum_v u^v(c_t^v, \phi^v(a_t)a_t^v) + \sum_\rho \psi_t^\rho(z_t^\rho - \mu K_t^\rho) + \sum_\sigma \psi_t^\sigma(z_t^\sigma - \mu K_t^\sigma),$$

where c_t^v is the vector of consumption, and z_t^ρ , z_t^σ are, respectively, the vectors of investment in the capital of private firm ρ and social institution σ , all at time t .

The optimum levels of consumption and investment at time t , c_t^v , z_t^ρ , z_t^σ , are obtained as the solution for the following maximum problem.

Maximum Problem. Maximize the imputed real national income in units of the utility at time t , H_t , subject to the feasibility constraints:

$$\sum_v c_t^v + \sum_\sigma c_t^\sigma \leq \sum_\rho x_t^\rho$$

$$\sum_v a_t^v + \sum_\rho a_t^\rho \leq a_t$$

$$a_t \leq \sum_\sigma a_t^\sigma$$

$$f^\rho(x_t^\rho, z_t^\rho, \varphi^\rho(a_t) a_t^\rho) \leq K_t^\rho$$

$$f^\sigma(a_t^\sigma, z_t^\sigma, c_t^\sigma) \leq K_t^\sigma,$$

where a_t^v , a_t^ρ are, respectively, the amounts of services of social common capital used by individuals v and private firms ρ , a_t^σ is the amount of services of social common capital provided by social institutions σ , and a_t is the total amount of services of social common capital, all at time t .

Let L_t be the Lagrangian form for this maximum problem:

$$L_t = \sum_v u^v(c_t^v, \varphi^v(a_t) a_t^v) + \sum_\rho \psi_t^\rho(z_t^\rho - \mu K_t^\rho) + \sum_\sigma \psi_t^\sigma(z_t^\sigma - \mu K_t^\sigma)$$

$$\begin{aligned}
& + p_t \left[\sum_{\rho} x_t^{\rho} - \sum_{\nu} c_t^{\nu} - \sum_{\sigma} c_t^{\sigma} \right] + \theta_t \left[a_t - \sum_{\nu} a_t^{\nu} - \sum_{\rho} a_t^{\rho} \right] + \pi_t \left[\sum_{\sigma} a_t^{\sigma} - a_t \right] \\
& + \sum_{\rho} r_t^{\rho} \left[K_t^{\rho} - f^{\rho}(x_t^{\rho}, z_t^{\rho}, \varphi^{\rho}(a_t) a_t^{\rho}) \right] + \sum_{\sigma} r_t^{\sigma} \left[K_t^{\sigma} - f^{\sigma}(a_t^{\sigma}, z_t^{\sigma}, c_t^{\sigma}) \right],
\end{aligned}$$

where $p_t, \theta_t, \pi_t, r_t^{\rho}, r_t^{\sigma}$ are, respectively, the Lagrangian unknowns associated with the constraints.

The optimum conditions are characterized by the following marginality conditions, in addition to the feasibility conditions:

$$u_{c_t^{\nu}}^{\nu}(c_t^{\nu}, \varphi^{\nu}(a_t) a_t^{\nu}) \leq p_t \quad (\text{mod. } c_t^{\nu})$$

$$u_{a_t^{\nu}}^{\nu}(c_t^{\nu}, \varphi^{\nu}(a_t) a_t^{\nu}) \varphi^{\nu}(a_t) \leq \theta_t \quad (\text{mod. } a_t^{\nu})$$

$$p_t \leq r_t^{\rho} f_{x_t^{\rho}}^{\rho}(x_t^{\rho}, z_t^{\rho}, \varphi^{\rho}(a_t) a_t^{\rho}) \quad (\text{mod. } x_t^{\rho})$$

$$\psi_t^{\rho} \leq r_t^{\rho} f_{z_t^{\rho}}^{\rho}(x_t^{\rho}, z_t^{\rho}, \varphi^{\rho}(a_t) a_t^{\rho}) \quad (\text{mod. } z_t^{\rho})$$

$$\theta_t \geq r_t^{\rho} [-f_{a_t^{\rho}}^{\rho}(x_t^{\rho}, z_t^{\rho}, \varphi^{\rho}(a_t) a_t^{\rho}) \varphi^{\rho}(a_t)] \quad (\text{mod. } a_t^{\rho})$$

$$f^{\rho}(x_t^{\rho}, z_t^{\rho}, \varphi^{\rho}(a_t) a_t^{\rho}) \leq K_t^{\rho} \quad (\text{mod. } r_t^{\rho})$$

$$\pi_t \leq r_t^{\sigma} f_{a_t^{\sigma}}^{\sigma}(a_t^{\sigma}, z_t^{\sigma}, x_t^{\sigma}) \quad (\text{mod. } a_t^{\sigma})$$

$$\psi_t^{\sigma} \leq r_t^{\sigma} f_{z_t^{\sigma}}^{\sigma}(a_t^{\sigma}, z_t^{\sigma}, x_t^{\sigma}) \quad (\text{mod. } z_t^{\sigma})$$

$$p_t \geq r_t^\sigma [-f_{x_t^\sigma}(a_t^\sigma, z_t^\sigma, x_t^\sigma)] \quad (\text{mod. } x_t^\sigma)$$

$$f^\sigma(a_t^\sigma, z_t^\sigma, x_t^\sigma) \leq K_t^\sigma \quad (\text{mod. } r_t^\sigma)$$

$$\theta_t - \pi_t = \tau_t \pi_t, \quad \tau_t = \frac{\tau(a_t) a_t}{1 - \tau(a_t) a_t}.$$

Lagrange unknowns p_t, θ_t, π_t may be interpreted, respectively, as the imputed prices of the output, the prices for the use of social common capital, and the prices paid for the provision of services of social common capital, and r_t^ρ, r_t^σ are, respectively, the imputed rents of capital in private firm ρ and social institution σ , whereas ψ_t^ρ and ψ_t^σ are, respectively, the imputed prices of real capital in private firm ρ and social institution σ , all at time t , measured in units of the utility.

A simple calculation shows that the imputed rents of capital in private firm ρ and social institution σ , r_t^ρ and r_t^σ , are, respectively, the marginal increases in total utility due to the marginal increases in the stock of capital in private firm ρ and social institution σ , both at time t . Hence, the sustainable processes of consumption and investment in the prototype model of social common capital may be obtained when the imputed prices of capital in private firm ρ and social institution σ at time t , ψ_t^ρ and ψ_t^σ are, respectively, equal to the discounted present values of the imputed rents of capital in private firm ρ and

social institution σ assuming that the imputed rents of capital remain stationary; i. e., the following relations hold for all private firms ρ and social institutions σ at all times t :

$$\psi_t^\rho = \frac{r_t^\rho}{\delta + \mu}, \quad \psi_t^\sigma = \frac{r_t^\sigma}{\delta + \mu}. \quad (9)$$

The relations (9) mean that the stationary expectations hypothesis holds true as regards the future schedule concerning marginal efficiency of investment of all kind of capital, private capital and social common capital.

Sustainable Processes of Capital Accumulation and Market Equilibrium

The optimum conditions for the sustainable processes at time t , as obtained above, are identical with those for market equilibrium at time t , when the imputed prices of capital in private firm ρ and social institution σ , ψ_t^ρ and ψ_t^σ are, respectively, regarded as the “market prices” of capital in private firm ρ and social institution σ , respectively, all at time t , assuming that the stationary expectations hypothesis holds true as regards the future marginal efficiency of investment of all kind of capital, and the social common capital taxes are levied on the use of services of social common capital.

Indeed, the optimum conditions above, together with the feasibility conditions, precisely correspond to the conditions for the market equilibrium in the prototype model of social common capital at time t :

(i) Each individual ν chooses the combination (c_t^ν, a_t^ν) of consumption c_t^ν and the use of services of social common capital a_t^ν so that the individual ν 's utility

$$u^\nu(c_t^\nu, \phi^\nu(a_t) a_t^\nu)$$

is maximized subject to the budget constraint

$$p_t c_t^\nu + \theta_t a_t^\nu = y_t^\nu,$$

where y_t^ν is the income of individual ν .

(ii) Each private firm ρ chooses the combination $(x_t^\rho, z_t^\rho, a_t^\rho)$ of production x_t^ρ , investment z_t^ρ , and the use of services of social common capital a_t^ρ in such a manner that net profits

$$p_t x_t^\rho + \psi_t^\rho z_t^\rho - \theta_t a_t^\rho$$

are maximized over $(x_t^\rho, z_t^\rho, a_t^\rho) \in T^\rho$.

(iii) Each social institution σ chooses the combination $(a_t^\sigma, z_t^\sigma, c_t^\sigma)$ of the provision of services of social common capital a_t^σ , investment z_t^σ , and the use of produced goods c_t^σ in

such a manner that net profits

$$\pi_t a_t^\sigma + \psi_t^\sigma z_t^\sigma - p_t c_t^\sigma$$

are maximized over $(a_t^\sigma, z_t^\sigma, c_t^\sigma) \in T^\sigma$.

(iv) At the prices p_t , total demand for goods are equal to total supply:

$$\sum_\nu c_t^\nu + \sum_\sigma c_t^\sigma = \sum_\rho x_t^\rho.$$

(v) At the prices for the provision and the use of services of social common capital, π_t and θ_t , the total amounts of the provision and use of services of social common capital are equal:

$$a_t = \sum_{\nu} a_t^{\nu} + \sum_{\rho} a_t^{\rho} = \sum_{\sigma} a_t^{\sigma}.$$

(vi) Social common capital taxes at the rate τ_t are levied upon the use of services of social common capital; i. e.

$$\theta_t - \pi_t = \tau_t \pi_t, \quad \tau_t = \frac{\tau(a_t) a_t}{1 - \tau(a_t) a_t}.$$

(vii) The expectations concerning future marginal productivity of capital of all kinds are stationary, i. e.,

$$\psi_t^{\rho} = \frac{r_t^{\rho}}{\delta + \mu}, \quad \psi_t^{\sigma} = \frac{r_t^{\sigma}}{\delta + \mu},$$

where ψ_t^{ρ} , ψ_t^{σ} and r_t^{ρ} , r_t^{σ} are respectively the imputed prices and the rental prices of the capital goods accumulated in private firm ρ and social institution σ .

The discussion above may be summarized as

Proposition. *In the prototype model of social common capital, the optimum conditions for the sustainable time-path of consumption and accumulation of private capital and social common capital coincide precisely with those for market equilibrium with the following*

assumptions:

(i) *The social common capital taxes at the rate τ are levied, i. e.,*

$$\theta - \pi = \tau\pi, \quad \tau = \frac{\tau(a)a}{1 - \tau(a)a}$$

where π , θ are, respectively, the price paid for the provision of services of social common capital and the price charged to services of common capital, and $\tau(a)$ is the impact coefficient with respect to the use of services of social common capital.

(ii) *The expectations concerning future marginal productivity of capital of all kinds are stationary.*

Concluding Remark

The sustainable processes of consumption and capital accumulation, including both private capital and social common capital, are obtained solely in terms of the state of the economy at each moment in time, independent of the state of the economy in the future.

On the other hands, the dynamically optimum processes of consumption and capital accumulation, including both private capital and social common capital, are obtained only under the hypothesis of perfect foresight concerning the future schedules of marginal efficiency of investment in all kinds of capital, as in detail discussed, e. g., in Uzawa (2003, 2005).

References

Mill, J. S. (1848) *Principles of Political Economy with Some of their Applications to Social Philosophy*, New York: D. Appleton [5th edition, 1899].

Uzawa, H. (1968) The Penrose effect and optimum growth. *Economic Studies Quarterly* 19, 1-14.

Uzawa, H. (1969) Time preference and the Penrose effect in a two-class model of economic growth. *Journal of Political Economy* 77, 628-652.

Uzawa, H. (2003) *Economic Theory and Global Warming*. New York: Cambridge University Press.

Uzawa, H. (2005) *Economic Analysis of Social Common Capital*. New York: Cambridge University Press.

Uzawa, H. (2009) Social common capital and sustainable development. *Macroeconomic Dynamics*.

Uzawa, H. (2010) Global warming, proportional carbon taxes, and International Fund for Atmospheric Stabilization. *Review of Development Economics*.