

# Optimal Property Rights in Financial Contracting

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## Abstract

In this paper we propose a theory of optimal property rights in a financial contracting setting. Following recent contributions in the property law literature (Merrill and Smith 2000, 2001; Hansmann and Kraakman 2002), we emphasize the distinction between contractual rights, that are only enforceable against the parties themselves, and property rights, that are also enforceable against third parties outside the contract. Our analysis starts with the following question: which contractual agreements should the law allow parties to enforce as property rights? Our proposed answer to this question is shaped by the overall objective of minimizing due diligence (reading) costs and investment distortions that follow from the inability of third-parties to costlessly observe pre-existing rights in a debtor's property. We argue that contracting parties can not fully internalize these costs, due to an inability to commit to protecting third-parties. Thus, the law might play an important role in minimizing transactions costs, not only by requiring adequate public notice to secure a property right, but also by refusing to enforce property rights that are likely to be redistributive in nature rather than in the interests of efficiency. We analyze examples of legal rules of this kind, including debtor-in-possession financing in bankruptcy; fraudulent conveyance laws;

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substantive consolidation; restrictions on the ability to enforce negative covenants as property rights; and limits on assignability.

## 1 Introduction

The economics of contracting literature and the legal scholarship conceive of property rights in very different ways. Economists, starting with the seminal contribution of Coase (1960), emphasize the role of property rights as a starting point for contracting. While the traditional statement of the “Coase theorem” stresses the irrelevance of the allocation of property rights for economic efficiency, later contributions by Alchian and Demsetz (1972) and Jensen and Meckling (1976) on the one hand argue that property rights can affect incentives, due to the status of the property owner as a residual claimant, and Williamson (1979) and Klein, Crawford and Alchian (1978) on the other argue that property rights provide protection against ex-post opportunism. Subsequently, Grossman and Hart (1986) and Hart and Moore (1990) have defined property rights instead as residual rights of control, and have shown that allocations of property rights can be valuable in alleviating holdup problems when contracts are incomplete. These formal analyses of property rights have been used to explain firm boundaries, capital structure, and authority relationships within organizations.

Although there may be differences in economists’ conceptions of property rights, economists usually start from the perspective that property rights are optimal allocations of rights *within a contracting coalition*. This perspective, and its implication for the role of the legal system, is important. When all affected parties start around a common bargaining table, as is often assumed in economic models, there is no role for a legal system beyond enforcing the contractual agreements reached by the parties. Left to their own devices, rational parties will be expected to allocate all relevant rights contractually, in a way that maximizes total social surplus.<sup>1</sup> As a result, the economist’s framework to date has little to offer in the way of a

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<sup>1</sup>The economists’ viewpoint often presupposes that the law is also necessary to defend an initial allocation of ownership rights to assets (however they may be determined), but this is not entirely obvious. Even if the law is completely silent on this issue and all assets are in the “public domain” at the outset, if all parties are available to bargain over the uses of assets going forward, efficiency is achievable. This implies that legal intervention in the realm of property rights (over and above enforcing contracts) is necessary only when third-parties outside the initial contracting coalition are affected.

positive analysis that explains features of property and contract law, nor does this framework offer normative prescriptions for the design of these laws, other than the recommendation that voluntary agreements should always be strictly enforced.

The economist's conception of property rights stands in sharp contrast to the concept of property in legal scholarship. This literature distinguishes property rights from ordinary contractual rights by defining property rights as rights *in rem* (rights to assets that are good against the world), while contractual rights are rights *in personam* (good only against the contracting parties themselves). In other words, property rights are unique because they bind not only the parties to a contract, but also bind third-parties *who lie outside a contracting coalition*.

The importance of this insight, highlighted in a series of recent law papers (Merrill and Smith 2000, 2001a, 2001b, Hansmann and Kraakman 2002) is that property law can play a more active role in increasing the efficiency of contractual agreements when third-parties outside a contracting coalition become relevant. When information about pre-existing rights is costly to acquire, these third-parties may be unknowingly affected by the rights of others. As a result, the law, as these authors argue, might optimally standardize the rights that can be created to limit externalities to unrelated parties (Merrill and Smith 2000)<sup>2</sup>, and it may also set limits on the notice required to make *in rem* rights enforceable (Hansmann and Kraakman 2002).

With this *in rem* feature of property in mind, we formally analyze the design of property laws in a financial contracting setting. We start with a firm run by an agent (call the agent *A*) that requires funding from two lenders, who each provide valuable capital to an investment project, but each lender contracts with the firm at a different point in time. As a result, the lenders may have competing claims to the firm's cash flows, and knowledge of the rights of pre-existing loan contracts may be imperfect.<sup>3</sup> The financial contracting

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<sup>2</sup>Merrill and Smith use the phrase "zone of privity" to include all parties that are relevant to a particular transaction, which would include P1, P2 and A in our setup. Unlike Merrill and Smith, who argue that all costs inside the zone of privity are internalized, we find that because of reading costs, P1 and A can not internalize the costs to P2, giving rise to a role for the law even in the absence of externalities outside the zone of privity.

<sup>3</sup>Our model assumes a sharp difference regarding the information about the contracts of other parties, which is costly to acquire, and the observability of one's own contract, which is assumed to be costlessly understood by the parties themselves. Thus, our model leaves room for legal intervention into property

context is a particularly important environment in which to consider these issues, because of the possibility that insolvency can result in incomplete satisfaction of a lender's claim. Thus, a mere *in personam* right to sue a bankrupt debtor can be substantially less valuable than an *in rem* right (such as priority rights to seize and sell collateral) that also binds past and/or future creditors. When the law allows for an early lender (call this lender  $P1$ ) to create stronger property-like protections, it can alleviate credit constraints by protecting  $P1$  against borrower moral hazard and the claims of a later lender (call this lender  $P2$ ). On the other hand,  $P2$  might act more conservatively in extending funds when he is uncertain about the pre-existing rights of  $P1$ . He might insist on being compensated for due diligence expenses to verify these pre-existing rights, and if he can not be sufficiently reassured, might forgo lending entirely.

Our model generates several findings. First, in a world without reading costs, there can be affirmative reasons for the law to allow  $A$  to grant  $P1$  not only seniority over  $P2$  (say, through a first-priority security interest in the final cash flow), but also to create an *in rem* right that voids any subsequent debt that  $A$  might incur. The right of  $A$  to create such *restraints on alienability* is valuable in a world in which monitoring  $A$ 's behavior is costly for  $P1$ , and  $A$  has the incentive to over-borrow from  $P2$  to continue his project inefficiently, thus *diluting* the value of  $P1$ 's claim. Intuitively, to ensure that his claim is repaid,  $P1$  may require not only seniority, but also that  $A$  retain sufficient cash flow rights so that his incentives to make the project succeed are preserved.

Given this affirmative justification for restraints on alienability (protecting earlier lenders from *dilutive contracts* by subsequent lenders), one might wonder why laws often limit the enforceability of these rights in practice. Our model suggests an answer when  $P2$  must expend reading costs to observe and fully understand the pre-existing rights of  $P1$ . If  $P1$  and  $A$  anticipate that  $P2$  will not conduct any costly due diligence to discover  $P1$ 's rights, this would open the door for  $P1$  to write a *redistributive contract* with  $A$  that diverts as much value from  $P2$  as the law will enforce. With this possibility in mind,  $P2$  will insist that  $A$  reimburses him for sufficient due diligence costs, enough so that  $P1$  and  $A$  will not be tempted to redistribute. In equilibrium, inefficient deadweight reading costs are incurred, and when these costs are sufficiently large, credit rationing to  $A$  may occur. Importantly, all rights, but not into contractual rights. Nevertheless, the argument of limited observability has been made in the contractual context; see Katz (1990).

of these deadweight costs are borne by  $A$  in equilibrium, but he cannot commit to eliminating them, because he cannot (in a costless, observable way) demonstrate to  $P2$  that he has not written a redistributive contract with  $P1$ .

We find that the law can (potentially) increase efficiency through two channels, both of which are observed in many features of current law. First, the law can refuse to enforce property rights that are more likely to be redistributive than efficient.<sup>4</sup> Some of these restrictions provide a “rule-out” of redistributive contracts by  $P1$ . Examples of these include fraudulent conveyance laws, which set aside contracts intended to “hinder, delay or defraud” creditors; and limitations on the ability to enforce negative covenants (such as limitations on future debt) as property rights. Other legal restrictions are of a “rule-in” variety, and provide protections to  $P2$  that override  $P1$ ’s contractual protections in circumstances when  $P2$ ’s actions are less likely to be dilutive to  $P1$ , such as priority for debtor-in-possession (DIP) lenders in bankruptcy.

As a second lever, the law can also increase efficiency by making due diligence more effective, by increasing the information about  $P1$ ’s rights available to  $P2$  at a lower cost. Examples of this include public notice registration systems, which not only publicize  $P1$ ’s rights, but often standardize them into a simple, lean form, as does the current Article 9 filing system for security interests in the Uniform Commercial Code (UCC). In addition to registries, the law also provides implicit minimum standards of notice to third-parties in order to enforce separation between legal entities, through substantive consolidation and veil-piercing remedies.

Our formal model adds two insights in the financial contracting context that differ from earlier work by Merrill and Smith (2000) and Hansmann and Kraakman (2002). First, we show that legal restrictions on property rights are valuable, not because they limit externalities *across firms* (i.e. an A-P1-P2 coalition increase due diligence costs for other A-P1-P2 coalitions), but because they reduce costs *within* a firm (i.e. A and P1 impose due diligence costs on P2, which A pays for in equilibrium). By reducing the temptation of P1 and A to redistribute value from P2, the law can reduce P2’s required due diligence costs, which A can not reduce on his own. Second, in a financial contracting context, an optimal law adds

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<sup>4</sup>The reader may wonder why the law should optimally refuse to enforce a right entirely, rather than requiring that P1 provide *actual notice* to P2 before enforcing it. In a world in which courts could cheaply verify that P2 was actually aware of P1’s rights, this would be true. If third-party verification of P2’s knowledge is imperfect and costly, though, mandatory limits on P1’s rights can be optimal.

more value by ruling out property rights that are the most *redistributive*, not necessarily rights that are the most idiosyncratic or fanciful, as these authors emphasize.

The rest of the paper will proceed as follows. Section 2 will introduce the general model and Section 3 solves for optimal contracts in a world where all information about pre-existing contracts is costlessly observable by third-parties. Section 4 solves the model in the presence of reading costs by third-parties, which leads to our key results regarding the optimal legal design of property rights and generates comparative statics that can be applied to existing features of the law. Section 5 discusses some of these features and how they relate to the principles in our model, and Section 6 concludes. Readers who are less interested in the formal model can proceed directly to the brief summary of the model and policy implications starting in Section 5.

## 2 Model

We consider a simple model of a firm with a single project that requires two rounds of financing from two different lenders. At date 1, a wealthless agent ( $A$ ) is endowed with a valuable idea, and must raise an amount of  $i_1$  from a principal ( $P1$ ) to start the project. To continue the project at date 2, the agent requires an additional cash input of  $i_2$  from a second principal ( $P2$ ). To focus on the interface between principal  $P1$ 's and  $P2$ 's claims, we shall make the restrictive assumption that  $P2$  can contribute no more than the required investment outlay  $i_2$  and that  $P1$  can not contribute the entire amount  $i_1 + i_2$ <sup>5</sup>. Also, both principals operate in competitive lending markets, all parties are assumed to be risk-neutral, and there is no discounting.

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<sup>5</sup>There may be several reasons why each principal is only willing to invest a limited amount. For one, the lenders may be wealth constrained, or they may prefer to have a limited exposure in a firm for risk-diversification reasons. Finally, principal  $P1$  may be reluctant to invest more than  $i_1$  for fear that the agent  $A$  simply wastes the surplus funds. It is possible to extend our model to allow for an endogenous determination of each principal's investment and to show that under some quite intuitive conditions each principal would not want to invest more than the required amount  $i_j$ . However, for the sake of simplicity and brevity we omit the discussion of this more general model.

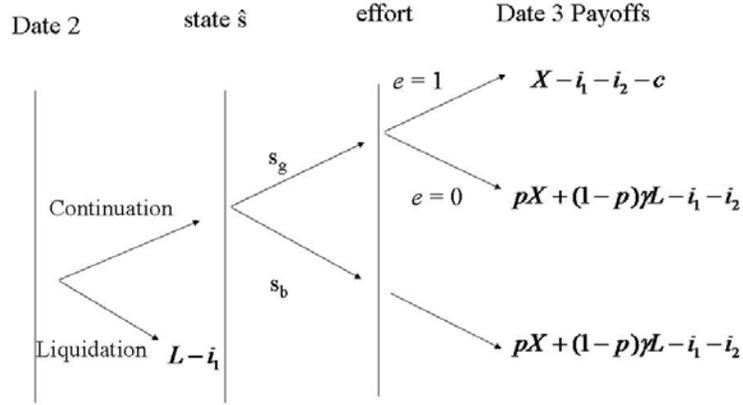


Figure 1

Figure 1:

## 2.1 Technological assumptions

If the project gets the two rounds of financing it produces a random cash flow at date 3. The cash flow outcome depends on the realization of the state of nature at date 2. We allow for only two states of nature,  $\hat{s} \in \{s_g, s_b\}$ . The good state of nature,  $s_g$ , occurs with probability  $\pi$  and the bad state,  $s_b$ , with probability  $1 - \pi$ .

In the bad state of nature the project yields a cash-flow of  $X$  at date 3 with probability  $p$  and with probability  $(1 - p)$  the project yields no cash flow but a liquidation value  $\gamma L$ , where  $\gamma < 1$ . In the good state of nature the cash-flow outcome of the project depends on the agent's effort choice  $e \in \{0, 1\}$  at date 2. If the agent chooses  $e = 1$  then the project yields a final cash flow  $X$  with certainty. If the agent chooses  $e = 0$ , the project yields the same cash-flow as in the bad state of nature. The agent's private cost of choosing high effort ( $e = 1$ ) is  $c > 0$ , and the cost of  $e = 0$  is normalized to zero.

If the project does not receive the required funding at date 2, it is liquidated for a known value  $L > 0$ .

We summarize the description of the project and its payoffs in Figure 1 above.

## 2.2 Contracting assumptions

The agent  $A$  and principal  $P1$  can write a bilateral long-term debt contract at date 1. Similarly, the agent and principal  $P2$  can write a bilateral debt contract at date 2. Each bilateral contract specifies the amount the principal agrees to lend  $i_j$  and a repayment  $F_j$  at date 3. The contract between  $P1$  and  $A$  can also specify a maximum amount  $\Phi_1$  of date 3 cash flows  $A$  is allowed to pledge to  $P2$ , and whether the claim  $F_1$  is senior, on par, or junior to  $F_2$ .

Importantly, our assumptions rule out the possibility that contract terms may be contingent on the state of nature  $s_l$ ,  $l = g, b$ . We justify this restriction on the usual grounds that the state of nature  $s_l$ , while observable to  $A$  and  $P2$  at date 2, is not verifiable in court.<sup>6</sup> We also rule out the possibility for now that  $P1$  is available to monitor the firm, or to renegotiate his contract with  $A$  at date 2 after the realization of the state of nature  $s_l$ . Thus,  $P1$  is a passive lender who can only lend at date 1 and collect at the final date. This assumption is admittedly strong, but is made to demonstrate in the simplest possible fashion the potential conflicts between  $P1$  and  $P2$  when they lend at different points in time.<sup>7</sup>

The four key economic issues in our contracting problem are as follows. First, the agent's repayment obligations  $F_j$  must be low enough that the agent has an incentive to put in high effort ( $e = 1$ ) in state  $s_g$ . Second,  $F_1$  must be sufficiently low to make room for continuation financing by  $P2$  at date 2, whenever continuation is efficient. Third,  $P1$  also faces a threat of dilution of the value of his claim  $F_1$  at date 2, when the agent issues a new claim  $F_2$  to  $P2$ . It is, of course, possible for  $P1$  to limit this dilution risk by issuing a senior claim  $F_1$ . However, as we show below, issuing a senior claim is not a sufficient protection against dilution in our setup. To obtain full protection  $P1$  must also specify a limit  $\Phi_1$  on date 3 cash flows the agent is allowed to pledge to  $P2$ . Fourth, and most importantly for our analysis, the very protections against the risk of dilution that  $P1$  specifies in his contract may, in turn, create a risk of loss for  $P2$ . This latter risk arises from the fact that  $P1$ 's contract with  $A$  may contain covenants that limit  $P2$ 's claims on the firm's cash flows, and the *due diligence* that

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<sup>6</sup>The non-verifiability of the state is not at all crucial to the results, but it simplifies the set of contracts that can be written.

<sup>7</sup>The assumption that  $P1$  is not available at all at date two becomes relevant because  $P1$  could potentially accelerate his loan in response to an attempt by  $P2$  to collude with  $A$  that would hurt  $P1$ . These issues clearly occur in reality, but they require that  $P1$  monitor  $A$  carefully, which comes at a cost, and a surprise acceleration of a loan might also be costly to  $P2$ .

$P2$  must expend to discover these covenants in the fine print of  $P1$ 's contract is costly and imperfect.

We model the due diligence costs  $P2$  faces at date 2 in a simple fashion. To read  $P1$ 's contract, we assume that  $P2$  must expend a due diligence (reading) cost  $\rho > 0$ . For modeling purposes, we shall make the extreme assumption that if  $P2$  does not spend this cost, he does not understand  $P1$ 's contract at all, but if he spends the reading cost  $\rho$  he understands the contract with probability

$$P(\rho) = \frac{\rho}{\rho + \kappa}$$

and does not understand it with probability  $(1 - P(\rho))$ , where  $\kappa \geq 0$ . In addition, we assume that the due diligence costs actually expended are private information to  $P2$  and thus not verifiable in court. The parameter  $\kappa$  thus allows us to capture in a simple way the difficulty in reading contracts. This setup is flexible enough to support several interpretations. One interpretation is that  $P2$  may find it costly to discover all of  $A$ 's existing contracts in the first place. Another is that  $\kappa$  represents the complexity of reading contracts once they are in the hands of  $P2$ . For idiosyncratic contracts and contracts that are particularly lengthy and detailed  $\kappa$  is likely to be high, while for standardized contracts with mainly boilerplate clauses, and for contracts that are more concise,  $\kappa$  would be low.

### **3 Optimal Contracting in a world with no information costs**

Economic models of contracting with multiple principals, similar to the one outlined above, are cast in a world where, i) there are no information costs; ii) there is freedom of contracting; iii) property rights are exogenously given; and, iv) contracts are perfectly enforced by courts<sup>8</sup>. What precise form property rights take in these models is typically not spelled out explicitly. It is helpful, therefore, to begin our discussion in this section by teasing out explicitly the underlying assumption on property rights in these models. We then proceed with an analysis of optimal contracts when there are no information costs.

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<sup>8</sup>See Bernheim and Whinston (1985, 1986), Segal (1999) and Bolton and Dewatripont (2005)

### 3.1 Legal rules: The Coasean environment

In this section we attempt to spell out the benchmark legal environment that has become standard in the economics literature. We refer to this environment as the *Coasean legal environment*. It has, in our view, the following three main components:

a) **Well-defined, fully-alienable, and fully-divisible property rights**

In our *common agency* setup,  $A$ 's initial endowment is his idea (and his human capital), and the principals  $P1$  and  $P2$  are endowed with their cash stocks, respectively  $i_1$  and  $i_2$ . The assumption on property rights is that these individuals begin at date one with *full ownership rights* to these assets and that these will be perfectly enforced by a court. Full ownership rights are defined as a bundle of *property rights* similar to the notions of *usus*, *fructus*, and *abusus* under Roman law:

Thus, the *full owner* of an asset has all of the following property rights:

- a) the exclusive right to use the asset (*usus*),
- b) the exclusive right to receive income from the asset (*fructus*),
- c) the exclusive right to modify or transform the asset (*abusus*).

Furthermore, we also single out among *abusus* rights,

- d) the exclusive right to transfer any subset of these rights by contract (*alienability*).

Thus, in the Coasean legal environment, full ownership is a starting point, and the bundle of property rights that comprise ownership can be freely divided.

b) **Freedom of Contracting:** Courts will enforce all contracts regarding transfers of property rights (based on information they can verify), with no restriction on the space of allowable contracts, other than that the property right being transferred must be under the initial ownership of one of the contracting parties. Note that this definition allows for parties to write enforceable contracts that place restraints on alienability. In the present context, for example, if  $A$  has the right to the cash flows from an asset  $X$ , she may retain the right to spend the cash, but she could also transfer to  $P1$  the right to sell these cash flows to a third-party or to pledge them as collateral for a debt contract.

c) **First-in-time (FT) rule:** in the Coasean legal environment, when any inconsistency arises between contracts, the first contract written will have priority.

We should emphasize the strength of the FT rule in the Coasean legal environment, which differs from most real-world laws of property and contract. To give a concrete example that will be relevant to our model, suppose  $A$  writes the following sequence of contracts with  $P1$

and P2:

$C_1$  : P1 will lend 45 dollars to A and is entitled to the first 50 dollars of the firm's final cash flow. Any subsequent claim on the firm's cash flow by any third-party is null and void.

$C_2$  : P2 will lend 25 dollars to A and is entitled to 30 dollars of the firm's final cash flow.

Now suppose that the final cash flow is 100. In the Coasean legal environment, P1 would receive 50, A would receive 50, and P2 would receive zero. In contract  $C_1$ , A transferred away his right to pledge future cash flows to subsequent lenders. Thus, the FT rule would require that P2's claim be voided; he would have no right to recover anything from A, even though the firm is solvent, A had knowledge of his inability to pledge cash flow to P2, and A receives a payout that would allow him to pay P2 in full.

### 3.2 Optimal Contracting with no reading costs

We shall restrict ourselves to a subset of parameter values for which the optimal contract for P1 and A, and for P2 and A, is such that continuation with high effort is optimal in the good state and liquidation at date 2 is optimal in the bad state.

For ease of exposition, we will use the notation  $R_g$  to denote the maximum pledgeable income to P1 in the good state, conditional on continuation with effort:

$$R_g \equiv X - \frac{c}{1-p} - i_2 \quad (1)$$

To see that this is the maximum pledgeable income to P1, note that in order to encourage A to choose high effort, A requires a sufficient stake  $w_g$  in the output when the project succeeds. An optimal contract will pay the agent  $w_g$  when the cash flow is  $X$  and 0 if output is 0. Thus, in order to elicit effort from A, the following incentive compatibility constraint must be satisfied:

$$w_g - c \geq pw_g$$

which reduces to

$$w_g \geq \frac{c}{1-p}.$$

Therefore, the maximum pledgeable income to all lenders is  $X - \frac{c}{1-p}$ . Since P2 will not participate unless he receives an expected payment equal to his monetary contribution, P2 must be repaid  $i_2$ . Thus the maximum pledgeable income to P1 is as in (1).

With this notation, the parameter restrictions we maintain throughout the paper are:

**Assumptions:**

**A1)**

$$X - c - i_2 > L$$

The first assumption tells us that in the good state, continuation with high effort is economically efficient relative to liquidation.

**A2)**

$$pX + (1 - p)\gamma L - i_2 < L$$

Assumption A2 says that continuation with low effort is inefficient relative to liquidation; hence liquidating the project will be optimal in the bad state at date 2. Assumptions A1 and A2 together imply also that high effort is efficient relative to low effort in the good state.

**A3)**

$$\pi R_g + (1 - \pi)L \geq i_1$$

Assumption A3 implies that the first-best action plan, which involves continuation in the good state with effort and liquidation in the bad state, can generate enough cash flow to repay  $P1$  for his loan. Since we assume that  $L < i_1$ , A3 also implies that  $R_g > L$ ; i.e. continuation with effort produces more pledgeable income to  $P1$  than liquidation in the good state.

Finally, we shall also assume that:

**A4)**

$$X - R_g \geq \frac{i_2}{p}.$$

Assumption **A4** implies that  $P1$  may be at risk of *dilution* of his claim in the bad state if he writes a debt contract with  $A$  where  $\Phi_1 = X$ . This assumption is central to our analysis, since it implies that  $P1$  will not be fully protected against the risk of dilution by seniority alone. In order to ensure that  $P1$  does not suffer from dilution at the hands of  $P2$  and  $A$ , he requires protection over and above the protection that seniority provides. We will demonstrate this more fully in the next section.

### 3.2.1 First-best outcome

Suppose a benevolent, social welfare-maximizing planner could observe the state of the world and make all investment and effort decisions. Under the assumptions above (**A1-A4**), the social planner would choose to fund the project, to continue the project in the good state at date 2 while at the same time choosing high effort ( $e = 1$ ), and to liquidate the project at date 2 in the bad state. This first-best action plan would maximize social welfare, which is given by

$$\pi(X - c - i_2) + (1 - \pi)L - i_1$$

### 3.2.2 Implementation: state-contingent contracts

If the contracting parties can write (bilateral) state-contingent contracts, then this first-best action plan can be implemented as a subgame perfect equilibrium (SPE) of the following contracting game.

At date 1, the agent makes the following take-it-or-leave-it offer of a state-contingent debt contract to  $P1$ . Agent  $A$  borrows  $i_1$  from  $P1$  and in exchange agrees to:

1. liquidate the project and to pay the entire liquidation proceeds  $L$  to  $P1$  at date 2 in the bad state, and
2. to repay  $P1$  a face value of debt

$$F_1 = \frac{i_1 - (1 - \pi)L}{\pi}$$

at date 3 in the good state, and finally

3. to make  $F_1$  senior to any subsequent claims on the firm.

Given that this contract covers  $P1$ 's investment  $i_1$  in expected terms,  $P1$ 's (weak) best response is to accept this contract.

It is easy to see that the best response to this contract for  $A$  in the good state at date 2 is to offer  $P2$  the following contract:  $A$  borrows  $i_2$  dollars from  $P2$  in exchange for a junior debt claim with face value  $F_2 = i_2$ . Again, as this contract covers  $P2$ 's investment  $i_2$ ,  $P2$ 's (weak) best response is to accept this contract.

Finally, to see that  $A$ 's contract offer at date 1 is a best response to the respective equilibrium moves of  $P1$  at date 1, and  $A$  and  $P2$  at date 2, observe that under this contract  $A$  gets the first-best expected payoff  $\pi(X - c - i_2) + (1 - \pi)L - i_1$  which is equal to total social welfare under the first-best action plan. This is the highest expected payoff  $A$  could achieve in any equilibrium, since any deviation from the first-best action plan at date 2, induced by another contract offer, would be anticipated by  $P1$  and priced into the loan contract through a higher  $F_1$  (i.e. a higher interest rate). In other words,  $A$ 's private objective is perfectly aligned with social welfare in a Coasean legal environment, and therefore  $A$ 's choice of contract implements the first-best social outcome.

### 3.2.3 Incomplete contracts: the insufficiency of seniority

While a first-best outcome is straightforward to implement under complete contracting, it is less obvious under incomplete contracting (when courts cannot observe the state of the world). At first glance, one might expect that a simple senior debt contract alone would be sufficient to generate the socially efficient outcome even with non-contingent debt contracts.<sup>9</sup>

Indeed, if  $P1$  has a senior debt claim one might expect that this would generate the right social incentives for  $P2$  to refuse to lend in the bad state, since he bears more of the cost of failure than  $P1$ <sup>10</sup>. Even so, under assumption **A4**, this is not the case. Since under assumption **A4** we have  $X > R_g + \frac{i_2}{p}$ , it is still in the joint interest of  $P2$  and  $A$  to continue the firm inefficiently at the expense of  $P1$ , and thus to *dilute* the value of  $P1$ 's debt claim. Indeed,  $P2$  is then willing to lend  $i_2$  and take a junior debt claim with face value  $F_2 = \frac{i_2}{p}$  and  $A$  would then receive an expected payoff from continuation of

$$p(X - F_1 - F_2) > p(X - R_g - \frac{i_2}{p}) > 0,$$

which is strictly higher than what  $A$  gets in liquidation.<sup>11</sup>

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<sup>9</sup>The idea that junior debt can be used to dilute senior claims in the presence of moral hazard was originally formalized in Bizer and DeMarzo (1992).

<sup>10</sup>Since  $P1$ 's loan is senior, he will recover the entire cash flow in the low state if the project fails,  $\gamma L$  while  $P2$  will receive nothing. Thus, the consequences of failure are more severe for  $P2$  than for  $P1$ .

<sup>11</sup>It is possible to correct this inefficiency by giving  $A$  a payment in the event of liquidation, of say  $\phi L$ , sufficient to offset the positive gain  $A$  would get under continuation. Deviations from absolute priority in bankruptcy could, thus, be rationalized in our model as a way of forestalling inefficient continuation.

In a somewhat richer model, however, one might be concerned that by structuring the agent's incentives

Thus, under the parameter assumptions in the model, seniority alone is not sufficient to protect  $P1$ . Though social welfare is destroyed by the inefficient continuation, the value transferred from  $P1$  to the  $P2/A$  coalition outweighs this loss when **A4** holds. Thus, the incentives of  $P2$  and  $A$  are not aligned with social welfare when a simple senior debt contract is written. Since  $A$  bears this efficiency loss in equilibrium,  $A$  would prefer to give  $P1$  stronger rights than seniority alone in order to achieve efficiency and maximize his private payoff. Giving an additional property right to  $P1$  to specify a limit  $\Phi_1$  of date 3 cash flows  $A$  is allowed to pledge to  $P2$  achieves this goal.

### 3.2.4 The value of restraints on alienability

In the good state  $P2$  is willing to lend  $i_2$  in exchange for debt with face value  $F_2 = i_2$ , since the project will succeed with certainty.<sup>12</sup> In the bad state, however, the project fails with probability  $1 - p$  if it is continued. As we have pointed out above,  $P2$  will then require a face value of debt higher than  $i_2$  ( $F_2$  must be at least  $\frac{i_2}{p}$ ) in order to be compensated for this added default risk. Thus, the following contract will result in a first-best outcome:

**Proposition 1** *Under assumptions **A1** to **A4**, an optimal contract between  $P1$  and  $A$  is such that  $A$  takes a loan  $i_1$  in return for a date 3 senior (collateralized) debt repayment of*

$$F_1 = \frac{i_1 - (1 - \pi)L}{\pi},$$

in this way one might undermine her incentives to perform at date 1. For example, if efficiency requires that  $A$  raise the probability of reaching the good state from  $\lambda$  to  $\pi > \lambda$  at date 1, by taking action  $a = 1$  with private effort-cost  $\psi$ , rather than the free action  $a = 0$ , then rewarding the agent in the event of liquidation might be counterproductive.

Indeed, the agent's incentive constraint at date 1 :

$$\pi(X - F_1 - F_2) - \psi \geq \lambda(X - F_1 - F_2)$$

without any payment in liquidation might be satisfied, while the constraint with a payment  $\phi L$  in liquidation :

$$\pi(X - F_1 - F_2) + (1 - \pi)\phi L - \psi \geq \lambda(X - F_1 - F_2) + (1 - \lambda)\phi L$$

might not.

<sup>12</sup>By definition of  $R_g$ , as long as  $P1$  is promised no more than this amount,  $P2$  can be promised  $i_2$  if the good state occurs, and  $A$  will prefer high effort. Therefore, the probability of success will be 1 and  $P2$  will be repaid with certainty.

and a commitment not to pledge more than  $\Phi_1 = i_2$  to  $P2$  at date 3.

The best response for  $P2$  and  $A$  at date 2, then is to sign a new loan contract only in the good state specifying a loan of  $i_2$  in return for an equal safe claim of  $i_2$  at date 3.

**Proof.** see appendix ■

In order to implement the first-best,  $P1$  requires not only seniority, but also that  $A$  make a credible commitment not to pledge new cash-flow in excess of  $i_2$ . This commitment can be achieved by transferring to  $P1$  the right to pledge the project's cash flows to future lenders over and above  $i_2$ . Thus,  $\Phi_1$  is a form of *negative covenant*, which would apply to all future debts in excess of  $i_2$ , whether these are secured or unsecured. Since  $P2$  understands that  $A$  can legally pledge no more than  $i_2$ , he is not willing to lend in the bad state, and the first-best is achievable by contract.<sup>13</sup>

It is also worthwhile to note the value that a *property right* (a right in rem) gives to  $P1$ , instead of a mere *contractual right* (a right in personam). Instead of the Coasean legal environment we consider here, suppose that  $A$  was not allowed to create restraints on alienability in its loan contract with  $P1$  that binds third-parties. Then if  $A$  breaches its contract with  $P1$  (by promising  $P2$  junior debt with face value greater than  $i_2$ ),  $P1$  would have a *contractual right* if it could sue  $A$  for breach of its contract, but could not invalidate  $P2$ 's debt. In this environment, contractual rights alone would not be sufficient to obtain the first-best outcome. If the breach is discovered after the loan is sunk (i.e. only by date 3),  $P1$  would have a claim for damages which would be junior to  $P2$ 's claim<sup>14</sup>. Anticipating this,  $P2$  will still be willing to lend, and the inefficient continuation will not be prevented.

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<sup>13</sup>Note that a very strong legal system is required here. If  $P2$  can take a claim on  $A$ 's personal assets (his dividend from the firm at the end of date three) then he would be equally happy to lend into an inefficient continuation in the bad state. Thus  $P1$ 's right to restrict alienability must extend beyond the corporate form and also to  $A$ 's assets more generally in order to effectively shut down  $P2$ 's loan.

<sup>14</sup>This would follow from the fact that  $P2$  has a property right in the project's cash flows. When  $P1$  wins his breach of contract suit, the property available to satisfy the judgment would be  $A$ 's property, which would be only the equity in the firm, but not  $P2$ 's claim.

This argument assumes, however, that  $P1$  would have no rights to sue  $P2$  as well for tortious interference with contract. If this were possible, the first-best could be achieved by this means. In such a world,  $P1$ 's right to prevent additional debt could again be considered a property right, since  $P1$  has a right that is good against a third-party rather than a right which is only good against  $A$ .

## 4 Equilibrium Contracting with Reading Costs

We have shown that in our model there are efficiency gains to be had by allowing firms to create restraints on their ability to alienate cash flow rights. Moreover, in a world with no transactions costs and perfectly observable contracts, there only benefits and no costs to these restraints. The limits on alienability merely allow the firm to commit to protecting early lenders against the ex-post risk of dilution at the hands of subsequent lenders. Thus, in a perfect world with fully observable contracts there are affirmative reasons to allow for such contracts to be enforceable. In this section, we introduce contract reading costs and show that limits on alienability also create costs for third parties. When alienability of assets can be restricted in any way contracting parties desire, it becomes more difficult and costly for third parties to determine which assets are alienable and under what contingencies. These reading costs third parties face are a form of negative externality that the contracting parties impose on others. What is more, the contracting parties are not well placed, as we shall show, to internalize these externalities.

### 4.1 The contracting game with reading costs

We begin this subsection with a description of the contracting game between  $A$ ,  $P1$ , and  $P2$ , and an intuitive description of the equilibrium. Readers who are interested in a more complete description (which includes a specification of equilibrium beliefs by  $P2$ ) can consult the technical appendix for more details.

Before negotiations between  $P2$  and  $A$  start,  $P2$  is now unable to observe the contract between  $P1$  and  $A$  (which we denote by  $C_1$ ) without incurring reading costs. Thus, when negotiations begin,  $P2$  can only form a prior belief over what type of contract  $P1$  and  $A$  have signed at date 1. As in standard signaling games,  $P2$  can, however, rationally revise his beliefs about the initial contract between  $P1$  and  $A$  when he sees  $A$ 's contract offer  $C_2$ .

We assume that the contracting game at date 2 then proceeds as follows:

1. Agent  $A$  begins by making a loan contract offer  $C_2 = \{i_2, F_2, \rho\}$  to  $P2$ , which contains the terms of the second loan,  $F_2$  (as well as its priority status in repayment at date 3) and also a commitment by agent  $A$  to also cover up to  $\rho$  dollars of  $P2$ 's due diligence costs.<sup>15</sup>

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<sup>15</sup>For simplicity, we assume that due diligence costs can be paid in-kind; that is,  $A$  can confirm that these

2. Principal  $P2$  can accept or reject this offer. If  $P2$  rejects the offer, the game is over and the firm is liquidated. If  $P2$  accepts, he proceeds with the due diligence specified in  $A$ 's contract offer. He will then observe and understand the contract between  $P1$  and  $A$  with probability  $P(\rho) = \frac{\rho}{\rho+\kappa}$  and with the complementary probability  $(1 - \frac{\rho}{\rho+\kappa})$ ,  $P2$  fails to understand the contract. Failure to understand amounts to not observing the contract at all.
3. Finally, after completing the due diligence  $P2$  decides whether or not to lend given the contract he has observed, or given his updated beliefs about  $C_1$ .

This simple setup is intended to capture the possibility that  $P1$  and  $A$  may have written into the contract protective covenants that have the effect of redistributing date 3 cash-flows to them rather than  $P2$ . The second lender's uncertainty can come from two possible sources: he may be unsure that he observes the entirety of the pre-existing loan contracts that  $A$  has written. For example, he may be wary that  $A$  did not disclose a hidden obligation, such as a loan guarantee to a parent company, that would reduce the assets available to  $P2$  in the event of default. Second, even if  $P2$  is confident that he possesses all relevant pre-existing contracts, some of the covenants in these contracts may be written in a language that is not obvious to decipher, or whose implications for  $P2$ 's rights are not obvious to ascertain. The parameter  $\kappa > 0$  then represents the difficulty of discovering the meaning or implications of a clause: as  $\kappa$  approaches zero, even low levels of due diligence will discover hidden terms with probability approaching one; as  $\kappa$  grows toward infinity, a given due diligence expenditure discovers hidden terms with probability approaching zero.

Although  $P2$  may not always discover a hidden term, he is rational in that he anticipates that he "may have missed something", and makes his lending decision given this risk.  $P2$  is aware, however, that the more due diligence that  $P1$  and  $A$  anticipate by  $P2$ , the less likely is the possibility that  $P1$  and  $A$  may have included an overly restrictive clause in  $C_1$ , since discovery of the clause by  $P2$  would preclude further lending and result in an inefficient liquidation. Thus, due diligence gives  $P2$  confidence to lend, even if it never results in complete certainty about  $P1$ 's contract.

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costs will be spent on due diligence (as opposed to being divertable by  $P2$ ).

## 4.2 Equilibrium Contracting and Due Diligence

We begin our analysis by pointing out that there does not exist a Bayes-Nash equilibrium of the game with reading costs, which implements the first-best outcome without any due diligence by  $P2$ . To see this point, suppose that  $P2$  simply follows the same lending policy as before without reading the details of the contract between  $P1$  and  $A$  and hoping that  $P1$  and  $A$  would have written the first-best contract described in proposition 1 (we will refer to this contract as the *efficient contract*  $C_1^{fb}$ ). Could the efficient contract between  $P1$  and  $A$  still be an equilibrium move in a world with reading costs? If so, then the presence of reading costs for third parties would not be a serious concern for welfare, as agents would simply continue to draft contracts as if they were in a transactions-cost free world and they would not have to worry about imposing negative externalities on others. However, as intuition suggests and as the next lemma establishes, when  $P1$  and  $A$  expect  $P2$  not to do any due diligence and to follow the efficient lending policy irrespective of what form their own contract takes, then their best response is to write a contract that involves maximal redistribution from  $P2$  to themselves (call this contract  $C_1^x$ ). Adding some additional notation, let  $V_x$  denote the joint continuation payoff to  $P1$  and  $A$  in the event that they write this maximally redistributive contract<sup>16</sup> and  $P2$  lends. Then we have the following lemma:

**Lemma 2** *Suppose that  $P2$  always accepts the contract  $C_2 = \{i_2, i_2, 0\}$  in the good state without incurring any due diligence costs. Then the best response for  $P1$  and  $A$  is to write a maximally redistributive contract  $C_1^x$  that takes the following form:*

*Principal  $P1$  agrees to lend  $i_1$  dollars to  $A$  in exchange for a senior debt claim with face value  $F_1 = \frac{i_1 - (1-\pi)L}{\pi}$  and a covenant that fully restricts alienability of future cash-flows ( $\Phi_1 = 0$ ). In the Coasean legal environment,  $P1$  and  $A$  would receive the maximum possible joint continuation payoff  $V_x = X - c$ .*

This lemma implies that in a world with costs of reading contracts, it will be impossible to avoid these costs completely, because this would increase the likelihood of opportunism by  $P1$  and  $A$ .

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<sup>16</sup>The maximally redistributive contract  $C_1$  would set  $\Phi_1 = 0$ , so that  $P1$  and  $A$  would be able to claim the entire cash-flow net of effort costs:  $(X - c)$ .

In principle, the law could even allow for negative  $\Phi_1$ , implying that  $P1$  could seize  $P2$ 's property (over and above  $i_2$ ) if  $P2$  makes a loan. In a world with no reading costs, there would be no loss in enforcing these extremely redistributive contracts, because  $P2$  would never sign them.

We now proceed to describe what we will term the *least-cost separating equilibrium* of the contracting game. As the prior lemma makes evident, there can not be an equilibrium that involves  $P2$  lending despite conducting no due diligence. We focus on solving for the equilibrium that requires the lowest amount of due diligence such that  $P2$  is willing to lend in equilibrium even when due diligence is (ex-post) uninformative. This will be the lowest amount of due diligence that reassures  $P2$  that  $P1$  and  $A$  have not attempted redistribution at  $P2$ 's expense.

Intuitively,  $P1$  and  $A$  will find one of two possible strategies optimal. One strategy is to write the *efficient contract*  $C_1^{fb}$  that would be optimal between  $P1$  and  $A$  if  $P2$  could observe their contract costlessly. While this strategy yields a lower joint payoff than a successful redistribution attempt, the advantage of such a contract is that  $P1$  and  $A$  will receive any expected surplus with certainty. If  $P1$  and  $A$  were to agree on this contract, followed by the same contract offer  $C_2 = \{i_2, i_2, \rho\}$  their joint continuation payoff would be the following:

$$X - i_2 - c$$

The other alternative is to write the *maximally redistributive contract*  $C_1^x$ , hoping that  $P2$  will not discover it. This contract would return a higher joint payoff  $V_x = X - c$  to the parties if  $P2$  lends, but will result in liquidation if  $P2$ 's due diligence uncovers the contract, which occurs with probability  $P(\rho) = \frac{\rho}{\rho + \kappa}$ . In this scenario, the expected joint continuation payoff of  $P1$  and  $A$  is

$$\left(\frac{\rho}{\rho + \kappa}\right) L + \left(1 - \frac{\rho}{\rho + \kappa}\right) V_x$$

With these expressions in hand, the following inequality tells us when  $P1$  and  $A$  will write the efficient contract, given optimal behavior by  $P2$ :

$$X - i_2 - c \geq \left(\frac{\rho}{\rho + \kappa}\right) L + \left(1 - \frac{\rho}{\rho + \kappa}\right) V_x \quad (2)$$

Therefore, the cut-off  $\rho^*$  is given by the solution  $\rho$  for which (8) holds as an equality:

$$\rho^* = \frac{\kappa\{V_x - (X - i_2 - c)\}}{X - i_2 - c - L} \quad (3)$$

In the Coasean legal environment (in which the law allows fully-flexible design of property rights), the expression reduces to the following:

$$\rho^* = \frac{\kappa i_2}{X - i_2 - c - L} \quad (4)$$

In the least-cost separating equilibrium, P1 and A must set aside  $\rho^*$  up-front to compensate P2 for his due diligence. The final step in implementing this equilibrium is to verify that, inclusive of these due diligence costs, P1 and A prefer to implement an equilibrium that involves P1 lending at date 1, and continuing with effort in the good state by borrowing from P2 (we will relax these assumptions in the next subsection). This requires a slightly modified assumption to reflect the presence of positive reading costs:

**A3b)**

$$\pi(R_g - \rho^*) + (1 - \pi)L \geq i_1$$

Under this assumption the project can feasibly repay P1 inclusive of P2's due diligence costs, which are paid only in the good state.

With these assumptions in hand, we summarize this subsection by describing fully the least-cost separating equilibrium in the following proposition:

**Proposition 3** *Under the assumptions above (A1, A2, A3b, A4), the least cost separating Bayes-Nash equilibrium of the lending game with reading costs is as follows:*

*At date 1, P1 and A agree on contract  $C_1^{fb}$  taking the following form:*

- 1. P1 lends  $i_1 + \rho^*$  to A. In turn, A invests  $i_1$  in the project and holds  $\rho^*$  until date 2;*
- 2. P1 obtains a senior debt claim of  $F_1 = \frac{i_1 + \rho^* - (1 - \pi)(L + \rho^*)}{\pi}$ , and a commitment not to pledge more than  $\Phi_1 = i_2$  to P2;*

*At date 2:*

- 1. A offers contract  $C_2 = \{i_2, i_2, \rho^*\}$  to P2 in the good state,*
- 2. P2 accepts the contract, undertakes the due diligence, and invests  $i_2$  in the firm,*
- 3. The agent A then chooses high effort ( $e = 1$ ) and the project yields  $X$  at date 3;*
- 4. In the bad state of nature P2 refuses to lend and the project is liquidated, paying  $L + \rho^*$  to P1.*

**Proof.** see appendix ■

In this equilibrium, since we have assumed (by assumption A3b) that  $\rho^*$  is not too large, the only inefficiency caused by the presence of reading costs for  $P2$  are the deadweight costs of due-diligence  $\rho^*$ . It is important to note that the direct costs of due diligence are not the only economically relevant costs to imperfect observability. When one relaxes assumption A3b, so that  $\pi(R_g - \rho^*) + (1 - \pi)L < i_1 \leq \pi R_g + (1 - \pi)L$ , then  $P1$  does not expect to be repaid his initial contribution, and refuses to lend. As a result, due diligence costs causes credit-rationing: firms that would otherwise receive funding under costless observability can not obtain an initial loan from  $P1$ .

Whether the deadweight costs are the reading costs actually expended, or the indirect costs of underinvestment in valuable projects, it is clear that these losses will be higher when  $\rho^*$  is higher. A casual examination of (9), then, gives the following comparative statics:

**Corollary 4** *Relative to the first-best world with no reading costs, the social welfare loss in a world with positive reading costs is greater when:*

1. *Due diligence expenditures are less effective (higher  $\kappa$ );*
2. *The net gains from redistribution to  $P1$  and  $A$  ( $V_x - (X - i_2 - c)$ ) are larger;*
3. *The net present value of  $P2$ 's loan ( $X - i_2 - c - L$ ) is smaller.*

**Proof.** These follow immediately from the definition of  $\rho^*$ . ■

These comparative statics are intuitive. The less effective is due diligence in finding a hidden term, the more cost must be expended to eliminate the redistribution threat. When the net gains from redistribution ( $V_x - (X - i_2 - c)$ ) are larger relative to the cost of being caught ( $X - i_2 - c - L$ ),  $P2$  must be able to catch a redistributive covenant with greater probability for  $P1$  and  $A$  to prefer to write an efficient contract rather than a maximally redistributive one.

#### 4.2.1 Optimal Property Rights with Omniscient Courts

Up to this point, we have assumed a legal environment (which we termed the Coasean legal environment), in which the law allows contracting parties maximum flexibility in designing property rights that the law will enforce. In the setting with costless observability, the

first-best action plan is possible in the Coasean environment, implying that no alternative legal rule can be preferred.

In a world with reading costs, however, the Coasean legal environment is not a welfare-maximizing legal rule. To see this, suppose a social planner can observe and condition legal rules on the same set of variables that the parties can contract upon. Then an optimal legal rule would limit the rights that A could grant to P1, to eliminate the risk of expropriation. With this risk eliminated, P2 will be free to lend without requiring due diligence.

**Lemma 5** *In a world with perfect, omniscient courts, an optimal legal rule modifies the Coasean legal environment by adding the following limitations on the space of enforceable rights:*

1. *A limit on A's indebtedness: A can promise P1 a face value of no more than  $F_1 = \frac{i_1 - (1-\pi)L}{\pi}$*
2. *A rule against excessive restraints on alienability: A and P1 can set  $\Phi$  no less than  $i_2$ .*

*In this modified legal environment, the first-best action plan can be implemented by the sequence of contracts in Proposition 1 with no reading costs expended by P2.*

**Proof.** Omitted. ■

The lemma is useful in that it demonstrates, at least in principle, that legal rules limiting the set of enforceable property rights can increase social welfare. Nevertheless, the obvious critique of the above lemma is that it would require an unrealistic level of knowledge by courts to implement successfully in practice. Given that firms vary along many dimensions that are unobservable, the optimal cap on  $F_1$  and  $\Phi$  will be firm-specific and difficult to identify precisely on a case-by-case basis. As a result, legal rules that limit the space of enforceable property rights in practice will be subject to a trade-off: stricter restrictions may reduce due diligence and credit rationing costs, but due to their imperfect design, tighter restrictions will impose costs on parties who would write these contracts even in a world of perfect observability.

#### 4.2.2 Optimal Property Rights with Imperfect Courts

To see this trade-off in our formal model, consider the following (imperfect) legal rule: at date 2, A may promise P2 up to  $i_2$  dollars that is senior to P1. If A writes this contract with

$P2$ , it will be enforced notwithstanding the terms of the contract between  $A$  and  $P1$ . This mandatory “rule-in” of  $P2$ ’s loan by a court is conceptually similar to current law regarding *debtor-in-possession (DIP) financing* in Chapter 11 bankruptcy, as we will discuss in more detail in Section 5.

To compare welfare (which is also  $A$ ’s expected payoff) under these two legal rules, note that total expected welfare in the least-cost separating equilibrium in the Coasean legal environment (assuming that  $P1$ ’s participation constraint is satisfied) is given by

$$\pi(X - c - \rho^*) + (1 - \pi)L - i_1 - i_2 \quad (5)$$

While investment efficiency is guaranteed in the Coasean legal environment (continuation with effort in the good state, and liquidation in the bad state), the deadweight due diligence costs  $\rho^*$  are incurred in equilibrium. Social welfare under the modified legal environment that “rules in” the new loan is the following:

$$\pi(X - c) + (1 - \pi)(pX + (1 - p)\gamma L) - i_1 - i_2 \quad (6)$$

If  $P2$  knows for sure that he will recover at least the value of his loan, he would be willing to lend at fair terms to  $A$  at date 2 without the need for any due diligence. But as we have seen, the cost of providing  $P2$  with a certain return is that  $P2$  and  $A$  have the incentive to invest and continue in the bad state of the world. Comparing social welfare in (5) and (6), as long as

$$\Delta = \pi\rho^* + (1 - \pi)(pX + (1 - p)\gamma L - L) > 0$$

the “rule-in” legal environment will be social welfare-improving relative to the Coasean environment. It is crucial to reinforce that this result is driven by the inability of  $P1$  and  $A$  to commit to protecting  $P2$  in a world that allows complete contractual freedom. If  $\Delta$  is positive, this implies that  $A$  would like to commit to offering  $P2$  the seniority that the law mandates because of the due diligence cost savings, even though this would result in an inefficient continuation in the bad state, which raises the interest rate that  $A$  must offer  $P1$ . Though  $A$  prefers this outcome, he can not achieve it in the Coasean environment. Any attempt to offer this “guaranteed seniority” to  $P2$  would not be credible unless accompanied by an offer to reimburse  $\rho^*$  in due diligence costs.  $P2$  is aware that, due to the first-in-time rule in the Coasean world,  $P1$ ’s contract could contain a term setting  $\Phi = 0$ , which would essentially nullify  $P2$ ’s contract. Thus,  $P2$  will react with suspicion to any proposal that does not include reimbursement of due diligence, and refuse to participate.

Our model suggests that, in a world where legal design and courts are imperfect, there is a difficult trade-off to resolve in the design of property laws in a financial contracting setting. While we can not resolve these trade-offs quantitatively, we can present some rough guidelines that are relevant for resolving this trade-off.

Suppose a term in a contract between P1 and A creates a property right for P1 that binds P2:

1. If the division of property rights would not be part of an optimal contract between P1 and A in a world with no reading costs, and refusing to enforce this term reduces  $V_x$ , then the law should refuse to enforce this division of rights.
2. Conversely, if the division of property rights is part of an optimal contract between P1 and A in a world with no reading costs, and refusing to enforce this term does not reduce  $V_x$ , then the law should always enforce this division of rights.

When these distinctions can not be made with certainty, a trade-off exists from a legal design standpoint. Broadly speaking, our model suggests that the law should be permissive in enforcing a property right against a third-party when the (expected) forgone efficiency gains from enforcing the right are greater than the (expected) due diligence cost savings, which vary with  $\rho^*$ . Since  $\rho^*$  depends on  $V_x$  and  $\kappa$ , this suggests that the law should take a more permissive approach if the right is unlikely to be redistributive from P2, or if the costs of discovering the right by P2 are small. In the next section, we demonstrate that some of these trade-offs guide the design of many legal rules in practice.

## 5 Legal Rules and Optimal Property Rights

**Summary of the model** We begin this section with a brief summary of the model. In

the contracting problem we have analyzed, an agent ( $A$ ) has an investment project that requires funding from two lenders (P1 and P2) that arrive at different points in time. Under the assumptions in our model, A and P1 write a contract to provide P1 with two important property rights to protect the value of his claim. The first is a first-priority right to the firm's final cash flow. The second is a right to limit the future debt that A can offer to the second lender (P2). From a legal design standpoint, if these property rights are costlessly

observable to P2, we find that an optimal legal rule will enforce both of these rights against P2. Enforcing these rights enhances efficiency, because it prevents A from continuing the firm inefficiently at date 2 when liquidation is optimal. This, in turn, allows A to borrow from P1 at the lowest possible interest rate and maximize the value of the project.

When P1's rights are costly to observe, however, the situation changes. P2 recognizes that P1 and A have the incentive to write a different contract that redistributes value from P2 if they anticipate that P2 may not observe it. Anticipating this, P2 requires that A reimburse sufficient due diligence expenditures before lending. Though A internalizes the costs of due diligence that P2 requires, this does not imply that legal rules are irrelevant, because A can not credibly reassure P2 that he and P1 have not written a redistributive contract. Legal rules that place restrictions on the set of enforceable property rights, then, can be welfare-improving and desirable for A, as it allows A to make a credible commitment to P2.

Our model suggests, in a general sense, that optimal property rights in a world with imperfect courts with limited information must be designed to strike a balance in protecting P1 against P2 and vice-versa. Placing stronger legal limitations on the *in rem* rights that P1 can enforce reduces deadweight costs of due diligence by P2 and can reduce credit rationing when  $\rho^*$  is large relative to the value of P2's loan. On the other hand, excessive limits on P1's rights can make him vulnerable to dilution by P2: as we have seen, if the law refuses to enforce restraints on alienability, P1 can not simultaneously encourage continuation with effort in the good state and prevent dilution of his claim through inefficient continuation in the bad state. We have shown that optimal laws are more permissive in enforcing P1's rights when P2's ability to discover this right is sufficiently low, and when the right is less likely to be redistributive.

With these general principles in mind, we now discuss some features of the law that are intended to accomplish these goals, noting the balance that the law attempts to create between earlier and later lenders. We should emphasize that we do not seek to argue in this paper that the legal rules we describe below, as they are currently applied, are optimal. Our goal is merely to demonstrate that in many features of existing law that involve property rights, a balance is struck between allowing for contractual freedom, and ruling out redistributive property rights that would increase required due diligence, or lead to credit rationing by cautious lenders.

## 5.1 Legal rules that limit redistribution

In this section, we discuss legal rules in the financial contracting arena that are intended to reduce the likelihood that one lender will be the victim of redistribution at the hands of another. Broadly, these rules can be placed into two categories: in some cases, laws “rule-out” certain property rights (usually created by an earlier lender) that are potentially (or actually) redistributive; in other cases, laws “rule in” property rights (usually by a later lender) that are unlikely to be dilutive to an earlier lender.

### 5.1.1 Legal rules that “rule-out”

**Limits on enforceability of covenants as property rights** Under U.S. law, a crucial distinction between unsecured creditors and secured creditors is that secured creditors have important property rights with respect to their collateral which unsecured creditors do not. For example, a security interest will continue in the collateral if the debtor sells it to a third-party<sup>17</sup>, and the secured creditor will have priority over subsequent secured creditors in the same collateral. On the other hand, UCC Article 9 also limits the enforceability of contractual provisions intended to create restraints on alienability that bind third-parties. For example, the UCC (§9-401(b)) explicitly states that an agreement between a debtor and a secured party that prohibits transfer of the debtor’s rights to a third-party can not prevent the transfer from taking effect.

Similarly, courts are generally reluctant to grant property-like protections for unsecured creditors with negative pledge covenants that are intended to prevent the firm from granting security to a new lender. If a covenant in a loan agreement is violated, the lender may declare a default and accelerate his loan, but can not void the rights given in violation of the covenant.<sup>18</sup> Notably, exceptions may occur if the lender has *actual knowledge* of the negative covenant when it is violated.<sup>19</sup> This is consistent with our model: if P1 can demonstrate that

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<sup>17</sup>Exceptions are made if the collateral is sold in the “ordinary course of business” (cite). This logic conforms with the spirit of our model, in that the costs of checking for ordinary customers would be high relative to the value of the good itself. Hence the law protects the third-party buyer at the expense of the early lender.

<sup>18</sup>The oldest known case on this subject is *Knott v. Shepherdstown Manufacturing*, 5 S.E. 266 (W. Va. 1888) in which the court denied an equitable lien to an unsecured creditor with a negative pledge clause, arguing that the breach of the negative covenant gave rise only to a claim for damages.

<sup>19</sup>Bjerre (1999) notes the risk that the court may grant the unsecured creditor with a negative pledge clause

he made P2 aware of a term in his contract (if  $\kappa = 0$ , or if courts can verify that  $P(\rho) = 1$ ) then there is no cost to enforcing P1's rights exactly as he intends. Importantly, however, the law generally does not require that P2 expend costs to discover the negative covenants, and do not hold them responsible for gathering the information from A.

The standard rationale for the different rights of a secured creditor and an unsecured creditor with a negative pledge is that Article 9 requires that security interests must be publicized by filing a financing statement in order to be effective against third-parties; this makes the secured lender's rights cheaply verifiable by a third-party, while negative covenants are not registered. Given that discovery of a security interest requires lower cost than discovering a negative covenant (in the parlance of our model, the due diligence process for discovering a security interests requires lower  $\rho$  than does discovery of negative covenants due to registration), the law provides stronger restrictions on the latter.

This does not fully resolve the issue from a normative standpoint, however. Bjerre (1999), for example, argues that Article 9 could be expanded to allow registration of negative pledge clauses (prohibitions on future secured debt), thus allowing them to bind third-parties. Pursuing this logic further, the law could allow *any* negative covenant to be publicized, including stronger covenants (such as the ones we model here through the  $\Phi$  parameter) that void any subsequent debt, secured or otherwise.

Our model can offer one possible justification for the different legal treatment of the two contracts; namely, that the redistributive potential of a negative covenant is greater than that of a secured loan, even if the due diligence costs of understanding the implications of the two contracts are the same. The reason is that in order to redistribute with senior debt alone, P1 and A must encumber the firm with more debt than is socially efficient. This increases the likelihood that the firm fails, and reduces the joint payoff of P1 and A from a redistribution attempt at P2's expense. When negative covenants are allowed, P1 and A can write a redistributive contract that elicits first-best effort by A, which increases their joint payoff.

To see this, recall that in the Coasean legal environment (in which negative covenants are enforceable as property rights), the net gain to P1 and A from the maximally redistributive an equitable lien, which would bind third parties only if they have knowledge of the covenant. Similarly, in the case *First Wyoming Bank v. Mudge* (748 P.2d 713 Wyo. 1988) the court found that a secured lender who knowingly violated a negative pledge clause was held liable for tortious interference with contract.

contract is

$$V_x - (X - i_2 - c) = X - c - (X - i_2 - c) = i_2$$

When redistribution in the Coasean environment is successful, the firm borrows  $i_2$ , the project continues with high effort and P2 is repaid nothing. Now suppose that negative covenants are not enforceable as in rem rights, as under current law. Then the maximally redistributive contract between P1 and A would be a "dividend recapitalization" that looks like the following:

1. P1 lends  $i_1 + Div$  to A, in exchange for a senior debt claim with face value  $X$ , where  $Div = pX + (1 - p)\gamma L - i_1$  is the amount that causes P1 to break even given his face value  $X$ ;
2. A invests  $i_1$  in the project, and pays himself  $Div$  as a cash dividend, which he spends at date 1.

If P2 does not discover the hidden debt burden and lends, he will similarly receive nothing at date 3, since all the cash flow has been pledged to P1 who is senior. But in this case,  $V_x = pX + (1 - p)\gamma L < X - c$ . In order to expropriate from P2, P1 and A must drown the firm in debt, which in turn would reduce A's incentive to take effort. Because expropriation is less profitable for P1 and A, (i.e.  $V_x$  is lower), they have less incentive to attempt it, which reduces P2's required due diligence expenditures. As a result, legal intervention to limit the in rem rights of secured debt is not as valuable as the gains from limiting the in rem rights of negative covenant holders.

**Fraudulent conveyance** The law of fraudulent transfers is directly aimed at eliminating the type of redistributive contracts we describe in the model. Under U.S. law, the Uniform Fraudulent Transfers Act (UFTA) gives an unsecured creditor two ways to avoid a redistributive action by a creditor. First, the creditor can establish *actual fraud*, which involves establishing actual intent to "hinder, delay or defraud" a creditor, or *constructive fraud*, which does not rely on establishing the debtor's intent but instead relies on the debtor's

financial condition<sup>20</sup>. Importantly, the UFTA allows creditors to demonstrate fraud to protect themselves against prior and later transactions (i.e. P1 can use it against P2 and vice versa). In our model, if P1 and A attempted the redistributive dividend recapitalization in the previous subsection, P2 may be able to attack it as a fraudulent conveyance.<sup>21</sup> Importantly, courts have refused to apply fraudulent conveyance law to protect future creditors when the transaction is easily observable to these creditors, such as a well-publicized leveraged buyout.<sup>22</sup>

The proper role for fraudulent conveyance law is a topic that has received substantial attention in existing legal scholarship. Baird and Jackson (1985) argue that creditors can use protective covenants to prevent fraudulent conveyances (such as a leveraged buyout that dilutes earlier unsecured creditors) voluntarily if they so choose, but under current law, firms can not “contract out” of fraudulent conveyance protection if courts apply it erroneously or over-broadly. We agree with this point in principle, depending on how the opt-out is achieved. Our model does not justify any mandatory restrictions on P1’s ability to limit *his own* rights that are good against P2 (or vice versa).<sup>23</sup> This suggests that an optimal fraudulent conveyance law could be a default rule that allows creditors to opt-out of the protection in their own loan contract.<sup>24</sup> However, our model can be used to explain why the

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<sup>20</sup>Constructive fraud can be established if the creditor can show that the debtor firm a) received less than reasonably equivalent value for the transfer, and b) that the debtor was in a precarious financial situation at the time of the transfer (Blum, 2004)

<sup>21</sup>Under the hypothetical above, P2 could potentially demonstrate actual fraud, which requires demonstrating intent. Alternatively, P2 could demonstrate constructive fraud, which would require that A (the firm) did not receive reasonably equivalent value in exchange for the transfer, and that the transfer left the debtor with insufficient capital, or that the debtor was about to incur debts with the actual or imputed intention of not paying them when due. (UFTA Sec. 4(a))

<sup>22</sup>As the court argued in *Kupetz v. Wolf*, 845 F.2d 842 (9th Cir. 1988):

“Because fraudulent conveyance statutes were designed to protect creditors from secret transactions by debtors, the same rules should not apply when the transaction is made public. Future creditors may not complain when they knew or could easily have found out about the transaction. This certainly appears to be the case in this particular LBO. The transaction was well-publicized and the Trustee has not claimed or presented evidence that any of the future creditors were not aware of Wolf & Vine’s financial dealings.”

<sup>23</sup>Moreover, in such a context, A would have every incentive to reveal this contractual term to P2, as it would result in more generous lending terms from P2. This is not true in the opposite case (where A and P1 restrict P2’s rights), as A has the incentive to disguise this information.

<sup>24</sup>This is true, of course, subject to the risk that a creditor may fail to observe or understand a term in

law might refuse to enforce a contract between  $A$  and  $P1$  that prevents  $P2$ 's right to seek the fraudulent conveyance remedy, as this would require  $P2$ 's investigation to discover a right that may be harmful to him. In this (limited) context, our model implies that there is a valid trade-off between the benefits of reducing due diligence expenditures and credit-rationing, and the costs of ineffective or incorrect enforcement of this standard by courts.

**Limitations on anti-assignment clauses** The UCC also strikes a balance between protection of early and late principals in the context of assignment clauses. For instance, a firm operating as a franchisee ( $A$ ) may desire to grant a security interest in his franchisee rights to a lender ( $P2$ ) as a means of obtaining cheaper credit, but the franchisor ( $P1$ ) may value the right to restrict who can become a franchisee. In a different context, a bank ( $A$ ) might wish to sell its rights to payment on a loan to an investor ( $P2$ ), but the borrower ( $P1$ ) may be concerned about who his creditors are in the future.

These applications are a slight departure from our model in the sense that the principals are not both lenders, but the underlying tension is similar. If the law allows complete contractual freedom between  $P1$  and  $A$  to limit  $A$ 's ability to assign his rights to  $P2$ , this could result in redistribution from an uninformed  $P2$  who attempts to acquire  $A$ 's rights, and later finds himself empty-handed. The possibility of this outcome would increase the required due diligence of potential  $P2$ 's before agreeing with  $A$ , and potentially limit the liquidity of these financial contracts in secondary markets if  $P2$  attempts to resell them.<sup>25</sup> On the other hand, limiting the scope of  $P1$  and  $A$  to create such restraints might hinder efficient contracting. For example, after making a loan to a borrower, a relationship bank might be tempted to assign a loan to a lender who would be unwilling to forgive minor covenant violations, simply because this "tough" lender is willing to buy the loan at a high price.<sup>26</sup>

Revised Article 9 resolves this tension in a way that is consistent with balancing the key themes in our model. Contracts for which  $P1$  and  $A$  are likely to benefit from restraints

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his own contract, as in Katz (1990).

<sup>25</sup>Some anecdotal evidence from Canada supports this feature of our model. In Quebec and Ontario, anti-assignment provisions are not part of the commercial code. As a result, Fingerhut (2006), in an article targeted at practicing lawyers, warns that "additional due diligence is called for when the collateral includes Quebec or Ontario receivables."

<sup>26</sup>Consistent with this logic, Guner (2007) finds that borrowers extract concessions from banks that are likely to sell loans through lower interest rates.

on alienability are allowed more freedom of contracting. For contracts covered by 9-406, which include “accounts” such as receivables, it is less likely that P1 (the customer/borrower) will derive substantial value from the identity of his creditor. Moreover, if a financier was required to examine each individual agreement between a firm and its customers, the resulting due diligence costs might be prohibitive. As a result, the law overrides any contractual term that prohibits assignment. For contracts covered by 9-408, which includes “general intangibles” such as franchise and licensing agreements, and sales of “payment intangibles” such as commercial loans, the law provides more deference to freedom of contract. While the law under both sections invalidates any attempt to restrict the transfer of A’s rights to payment, under 9-408, the law allows P1 and A to create restraints on P2’s enforcement rights against P1. Thus, a borrower in a commercial lending context can ensure that he will not be subject to the aggressive collection tactics of an unknown loan buyer. Under 9-406, the law takes the extra step of invalidating even these types of contractual provisions.<sup>27</sup>

### 5.1.2 Legal rules that “rule-in”

**Debtor-in-possession financing and the automatic stay** Perhaps the most conspicuous example of legal intervention in financial contracting is when a firm files for bankruptcy. Under the U.S. bankruptcy code, the collection rights of earlier lenders which would exist outside of bankruptcy are prevented by the *automatic stay*. The firm is allowed to raise new money in the form of *debtor-in-possession (DIP) financing*, which is intended to keep the firm liquid and allow it to continue operations while a plan of reorganization is formed.

The relative priority of the DIP lender with respect to the firm’s existing creditors is determined by the standards set out in §364, as interpreted by the bankruptcy judge. The primary advantage to the firm from DIP financing is not merely that it allows the firm to issue senior (secured) claims, since firms have this ability outside of bankruptcy. As Triantis (1992) notes, the main advantage of a bankruptcy filing to the firm is the weakening of pre-bankruptcy contractual constraints imposed by debt covenants, such as acceleration clauses in the event of default. Lenders’ contractual protections are replaced by standards in the bankruptcy code, such as “adequate protection”, and the discretion of the bankruptcy judge, which is intended to protect secured creditors from dilution by DIP lenders.

As prior research has noted, many provisions of the bankruptcy code can be justified

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<sup>27</sup>For a thorough discussion of these issues, see Morse (2001), Plank (2001), and Schwarcz (1999)

as optimal default rules, but whether rules should be mandatory has been a topic of open debate.<sup>28</sup> In a world with no reading costs, our model suggests that there would be no harm to allowing complete “contracting out” of the bankruptcy procedure: if a firm expected an automatic stay and the right to DIP financing to be welfare-maximizing, it would have the proper incentive to contract for its use with its lenders.<sup>29</sup>

A world with reading costs can provide one explanation for why a mandatory rule can be optimal, but only if these costs are sufficiently high relative to the efficiency gains from allowing flexibility with respect to the choice of procedure. This, in turn, suggests that the means by which creditors would be allowed to “opt-out” is an important feature of these proposals. If early lenders are individually free to contract out of the automatic stay, and/or to limit the priority of the DIP lender through covenants in their loan contracts, the prospective DIP lender would need to thoroughly examine every outstanding loan contract to ensure that no such limiting covenants existed in each of them. At a time when the firm is severely liquidity-constrained and time is of the essence, the role of the law in limiting costly due diligence can be particularly valuable. The logic in our model suggests that allowing opting-out of the bankruptcy code should meet a minimum standard of observability to future lenders, such that discovery of the opt-out behavior can be made at sufficiently low cost.<sup>30</sup>

The comparative statics in our model suggest other reasons why information costs are particularly severe in bankruptcy. First, given that the firm is in financial distress, pre-bankruptcy lenders are aware that the firm’s going concern value  $X - i_2 - c - L$  is likely to

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<sup>28</sup>A summary of contract-theoretic alternatives to mandatory bankruptcy laws and prevailing counter-arguments for mandatory laws is beyond the scope of this paper; examples include Adler (2003), Bebchuk (1988), and Rasmussen (1992). A summary of the debate can be found in Warren and Westbrook (2005). Some of these counter-arguments defending mandatory laws require imperfect information on the part of creditors to operate.

<sup>29</sup>In the world with no reading costs, P1 and A would not contract to subordinate P1’s loan to P2 because this would lead to inefficient continuation in the bad state. DIP financing could implement the first-best action plan only if the bankruptcy judge could identify the good state and allow the senior loan only in that state.

<sup>30</sup>For example, Rasmussen (1992) proposes a menu-based approach as a means for allowing firms to tailor their choice of bankruptcy procedure, which they would announce by including in their corporate charter, and a unanimous vote would be required by creditors to modify it. Such a proposal, if implemented, would likely be easier for a potential lender to observe than opt-out provisions in the individual contracts of lenders.

be low. Our model suggests that in these circumstances, lenders to a firm in financial distress are rationally more concerned with contracting for an advantage over subsequent lenders, and less concerned about the risk of inefficient underinvestment. Thus, the likelihood of redistributive contracts is higher, requiring greater due diligence costs to be expended by a DIP lender. Finally, it is also likely that  $\kappa$  is particularly high in bankruptcy, since pre-bankruptcy lenders are likely to write more tailored contracts with more severe covenants that limit A's behavior. A contract that is more laden with idiosyncratic and detailed covenants may reduce the efficiency of P2's due diligence, which also tilts the scales toward protecting a new lender.

## 5.2 Legal rules intended to lower due diligence costs

### 5.2.1 Registries/notice-filing systems

Registries and other notice-filing systems provide a mechanism whereby P1 can make his rights available to P2 at a reasonably low cost. Under the UCC, providing notice to third-parties is, in most cases, a necessary condition for *perfection* of a security interest in an asset.<sup>31</sup> In other words, for the law to enforce P1's rights as property rights that are good against P2 as well as against A, P2 must have a means of discovering P1's rights at a sufficiently low cost. A notice-filing system provides a straightforward means of reducing  $\kappa$ , because P2 can be assured that the absence of any prior security interests in an asset means that P2 will be first in priority if it lends against the asset. Without such a system, P2 must rely on the honesty of A to reveal P1's prior interests, which he may have incentive to conceal.<sup>32</sup> Thus, for a given due diligence expenditure by P2, he can be more certain that he has discovered the existence of pre-existing rights that may be redistributive.

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<sup>31</sup>Under revised Article 9, sales of "payment intangibles" are automatically perfected and thus do not require notice filing. Schwarcz (2006) argues that this poses a problem for securitization of such assets, as potential buyers of these intangibles can not be certain about their priority status with respect to potential competing interests.

<sup>32</sup>Notice-filing systems are by no means universal around the world; Germany, for example, does not have a system to publicize non-possessory security interests and creditors must rely on the debtor's honesty. Schwarcz (1999b) advocates a universal system of registration for transfers of receivables as a means of reducing uncertainty in such transactions.

Many transition economies in Europe are in the process of implementing laws that support secured lending, and support agencies such as the European Bank for Reconstruction and Development (EBRD) and the U.N. Commission on International Trade Law (UNCITRAL) have developed model laws to assist countries in implementing these changes. Both organizations recommend that countries create and maintain credit registries to publicize property rights in movable property, and these model laws advocate making registration a necessary condition for enforcing security interests against third-parties. Interestingly, both organizations also recommend that credit registries use standardized, simplified forms and present only necessary information (such as the name of the debtor, the name of the secured party, and the collateral) rather than allowing parties to post entire security agreements in a bulletin-board type format. The mandated standardization is recommended explicitly to reduce the required reading costs of third-party lenders.<sup>33</sup>

Though registries are simple, relatively inexpensive means of verifying pre-existing interests in a debtor's property (reducing  $\kappa$ ), they are generally not seen as a panacea for information problems between creditors in practice. First, in many legal environments, these systems can be complicated, cumbersome to use, or decentralized, requiring checks in many different locations to determine pre-existing interests with certainty.<sup>34</sup> Second, since they do not contain all relevant information to a potential lender, the lender must conduct further due diligence to determine the amount of pre-existing debt, the covenants that determine default, and its potential consequences of a default, all of which could affect the value of a new lender's claim. Normatively, since  $\kappa$  will rarely be zero in practice, there is some scope for ruling out redistributive rights even in the presence of registries; but a simple corresponding implication stemming from our model is that as the efficiency of the registration system improves, the law should expand the set of property rights that are enforceable.

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<sup>33</sup>The EBRD's "Guiding Principles for the Development of a Charges Registry" (2004) states the following: "The charges register should provide accurate and concise information in a form that is standard, simple, and understandable to any person searching the register without reference to other documents...There is danger of including too much information. Registering thirty pages of annexes from a legal agreement may satisfy lawyers, but will make the register undesirably opaque for the public." (p. 9-10)

<sup>34</sup>For concrete examples, see the specific country studies of secured transactions law reported by the Centre for Economic Analysis of Law at <[www.ceal.org](http://www.ceal.org)>

### 5.2.2 Substantive consolidation and veil-piercing

While a registry is a publicly-provided mechanism for increasing the effectiveness of due diligence, in other cases the law places a burden on the parties themselves (P1 and A) to make their rights sufficiently observable as a pre-condition to their enforcement. Though we have focused on seniority of claims *within a firm* as a means of protecting P1's claims against dilution by P2, another means of protecting P1 is through the creation of separate legal entities. For example, A might create a parent company and a wholly-owned subsidiary, and allow P1 to lend at the parent level, while P2 lends at the subsidiary level. This would imply that P2 would be senior to P1 with respect to assets held at the subsidiary level, but P2 would have no ability to reach the assets at the parent level if the subsidiary's assets are not sufficient to repay P2.

When such multi-tiered organizational structures exist, P2's information about which entity owns which assets, and the nature of the relationship between the two entities, is obviously important. As we have seen, A might have an incentive, for example, to disguise the fact that P2 is lending to an under-capitalized subsidiary rather than a well-capitalized parent company. When such misrepresentation is possible due to vague boundaries between entities, creditors can pursue a variety of remedies, depending on the circumstances. *Veil-piercing* is a legal remedy that unwinds limited liability, which can be used by a creditor of a subsidiary to reach assets owned by its parent company.<sup>35</sup> While the application of veil-piercing by courts is difficult to generalize, common reasoning includes misrepresentation by the firm about the subsidiary's finances or prospects of the creditor collecting against the parent, as opposed to the subsidiary alone. The "undercapitalization" of the subsidiary is often cited as reasoning in favor of piercing the corporate veil as well (Thompson 1991). Easterbrook and Fischel (1985) argues, in the same spirit as our model, that allowing for veil-piercing in these contexts can be understood as a means of providing incentives for firms to disclose their undercapitalization to creditors when a full investigation of the firm's finances is prohibitively costly.

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<sup>35</sup>Veil-piercing is an equitable doctrine that is determined on a case-by-case basis, but most commentators summarize that it is a remedy to be applied only in cases of fraud or approaching fraud: "the separate personality of the corporation will be disregarded or the corporate veil pierced whenever the separateness of the corporate form is employed to evade an existing obligation, circumvent a statute, perpetuate a fraud or crime or generally commit an injustice or gain an unfair advantage."

In a related context, *substantive consolidation* is a bankruptcy remedy that involves collapsing legal boundaries and pooling the assets and liabilities of related legal entities, and can be seen as a multi-party version of veil-piercing (Skeel 2004). Similar to the veil piercing context, courts have created standards in some cases that depend (in part) on whether the company disregarded the separateness of the related entities, so that creditors dealt with the firm as a single entity when extending credit. Proponents might argue successfully that consolidation is necessary to remedy harm caused to creditors who thought they were lending to a single entity.<sup>36</sup>This, in turn, gives the parties who would benefit from the separation of legal entities incentive to make these boundaries clear and observable to third-parties so that their priority will be upheld.<sup>37</sup>

## 6 Conclusion

In this paper, we adopt a definition of property rights that departs from most of the economics literature on the subject and follows the definition of property rights that prevails in legal scholarship. Because this definition emphasizes that property rights are rights that bind third-parties, a key issue surrounding property laws is that these third-parties may be imperfectly informed about the pre-existing rights that affect them. In a financial contracting context, these concepts are particularly important because borrowers may become insolvent. As a result, lenders are particularly concerned with contracting for rights that bind other lenders with competing claims.

We develop a formal theoretical model in which lenders and borrowers are rational, in that they anticipate the strategic behavior of other players, and can write sophisticated contracts that attempt to mitigate inefficient, opportunistic behavior. The model demonstrates that, in a world with costless and complete information, a legal environment that allows parties maximum flexibility to create and enforce any allocation of divided property rights is optimal. When observability is costly, however, there can be a role for the legal system to limit the space of property rights that are enforceable.

In a world with full enforceability, third-parties will not participate without conducting sufficient due diligence to reassure themselves that redistribution at their expense has not

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<sup>36</sup>See the discussion regarding *In re Auto-Train Corp* (810 F.2d 270, 276) in Baird (2005), p. 8-9.

<sup>37</sup>For two well-written summaries of major cases involving substantive consolidation and normative recommendations as to its appropriate scope, see Kors (1998) and Amara and Kolod (2006)

occurred. In equilibrium, these deadweight costs of due diligence are borne by the borrowing firms. Importantly, though, this does not rule out a role for optimal design of property laws. In our model, there is no way for firms to reduce these costs, due to an inability to commit to protecting third-parties from redistribution. The law can add value by providing firms with a credible mechanism to make this commitment. If the cost of discovering a right is large enough, and the right is potentially redistributive, then the law will optimally refuse to enforce such a right. The law in our model can be seen as mandatory, in that the law will mandate a relationship between the enforceability of a right and the cost of discovering that right by third-parties. On the other hand, if contracting parties can demonstrate to a court that they made third-parties aware of their pre-existing rights, then our model suggests they should be enforced.

In our investigation into existing law, we find several examples that broadly confirm the qualitative trade-offs in the model. Laws that govern financial contracting in which third-parties are affected often limit the ability of early lenders to create enforceable property rights that can be redistributive. These law is less likely to enforce a property right when it is unlikely that the right has an efficiency rationale, and is more likely to enforce the right when knowledge about the right is relatively inexpensive for a third-party to acquire.

While our formal model is intended to add an additional element of realism to the study of legal design in a financial contracting setting, there are other important factors our analysis does not address that are important. For instance, many of the mandatory standards in the law that are intended to protect third-parties also entail substantial ex-post litigation costs. In a world with costly courts that make judgment errors, later lenders could threaten to use the legal protections we document above in an opportunistic way as a means of extracting value from earlier lenders. This could lead to deadweight costs and inefficient allocations as a result, tipping the scales toward a more permissive legal environment. On the other hand, the ability of the early lender to protect himself by monitoring the firm's contracting with the later lender is not present in the current model. Adding the possibility of costly monitoring would imply that  $P1$  has other means of protecting himself from dilution by  $P2$ , reducing the cost of less-permissive legal rules.

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## 7 Appendix

**Proposition 6** *Under assumptions **A1** to **A4**, an optimal contract between P1 and A is such that A takes a loan  $i_1$  in return for a date 3 senior (collateralized) debt repayment of*

$$F_1 = \frac{i_1 - (1 - \pi)L}{\pi},$$

*and a commitment not to pledge more than  $\Phi_1 = i_2$  to P2 at date 3.*

*The best response for P2 and A at date 2, then is to sign a new loan contract only in the good state specifying a loan of  $i_2$  in return for an equal safe claim of  $i_2$  at date 3.*

**Proof.** Note first that under the contract written between  $P1$  and  $A$ ,  $P2$  is not willing to lend to  $A$  at date 2 in the bad state. By lending  $i_2$  principal  $P2$  gets an expected repayment which is less than the loan  $i_2$ . Indeed, the most  $P2$  can hope to get is

$$pi_2 + (1 - p) \max\{0, \gamma L - F_1\} = pi_2$$

since

$$\begin{aligned} \gamma L - F_1 &= \gamma L - \frac{i_1 - (1 - \pi)L}{\pi} = \\ \frac{\pi\gamma L + (1 - \pi)L - i_1}{\pi} &< \frac{L - i_1}{\pi} < 0. \end{aligned}$$

Next,  $P2$  is willing to lend to  $A$  at date 2 in the good state under the contract written between  $P1$  and  $A$ , since  $X - (i_2 + F_1) > \frac{c}{1-p}$ , or

$$\pi(X - \frac{c}{1-p} - i_2) - (1 - \pi)L \geq i_1$$

by assumption **A3**. And when  $X - (i_2 + F_1) > \frac{c}{1-p}$ ,  $A$ 's best response is to choose high effort ( $e = 1$ ), since then:

$$X - (i_2 + F_1) - c > p(X - (i_2 + F_1)) \tag{7}$$

as

$$F_1 \leq R_g \equiv X - \frac{c}{1-p} - i_2$$

by assumption **A3**, and by definition of  $R_g$ ,

$$X - R_g \geq \frac{c}{1-p}.$$

The RHS of (7) is  $A$ 's expected payoff under the low effort choice ( $e = 0$ ), since when the project fails and only yields a liquidation value  $\gamma L$  the firm's total liabilities ( $i_2 + F_1$ ) exceed its assets  $\gamma L$ , so that  $A$  gets zero.

**Proof.** When  $A$  chooses high effort the firm gets a cash flow of  $X$  for sure at date 3. The firm's debt is therefore safe, so that  $P1$  is willing to lend  $i_1$  in return for a debt repayment of the same amount at date 3. ■ ■

## 7.1 Technical Appendix: Equilibrium with Reading Costs

Before negotiations between  $P2$  and  $A$  start,  $P2$  is now unable to observe the contract between  $P1$  and  $A$ , which we denote by  $C_1$ . Thus, when negotiations begin,  $P2$  can only form a prior belief  $\nu(C_1) \in [0, 1]$  over what type of contract  $P1$  and  $A$  have signed at date 1. As in standard signaling games,  $P2$  can, however, rationally revise his beliefs about the initial contract between  $P1$  and  $A$  when he sees  $A$ 's contract offer  $C_2$  to  $\nu_2(C_1 | C_2)$ .

Unlike in standard signaling games, the game we have described above allows for the possibility that the uninformed party may become informed (at a cost). Hence, the game needs to specify how  $P2$  responds when he becomes informed about  $C_1$ . In the event that  $P2$  remains uninformed, however, our contracting game is played out in an entirely analogous way to a standard signaling game.

Namely, an equilibrium of our game is taken to be a *Bayes-Nash equilibrium*, where:

1. All agents play a best response given their beliefs, and
2. All players' updated beliefs are consistent with all agents' best responses.

Concretely, in a *Bayes-Nash equilibrium*:

- a  $P1$  and  $A$  choose  $C_1$  at date 1 given  $P2$ 's expected equilibrium best response,
- b  $A$  chooses the contract offer  $C_2$  optimally at date 2 given the past choice of  $C_1$  at date 1 and given  $P2$ 's beliefs  $\nu_2(C_1 | C_2)$ ,
- c  $P2$  best responds by rejecting or accepting the contract  $C_2$  (before and after completing the due diligence), and
- d  $P2$ 's beliefs  $\nu_2(C_1 | C_2)$  are consistent with the equilibrium choices,  $C_1$  and  $C_2$ .

To be able to determine how the contracting parties will play this game with reading costs we still need to establish what their final payoffs are under each play of the game. We shall assume that contracts continue to be enforced by the same rules as in the Coasean world. In particular, in the event where the final date 3 cash-flow outcome is  $X$ , but where  $P2$ 's claim on the firm,  $F_2$ , exceeds the commitment in contract  $C_1$  not to pledge more than  $\Phi_1$ , we continue to assume that  $P2$  cannot be repaid more than  $\Phi_1$ . More generally, we shall

assume that any seniority, negative pledge, or other limit on alienability of cash-flows that  $P1$  and  $A$  write into  $C_1$  is strictly enforced by courts.

We now solve for a natural Bayes-Nash equilibrium of the contracting game, which we shall take to be the focal equilibrium outcome of our contracting game. As is well known, the set of possible Bayes-Nash equilibrium outcomes in a signaling game is typically large and our game is no exception. This multiplicity is driven by the general form the conditional belief function  $\nu_2(C_1 | C_2)$  can take and the weak restrictions imposed by equilibrium consistency of beliefs requirement in a Bayes-Nash equilibrium. However, in our game as in other signaling games a particular belief function  $\nu_2(C_1 | C_2)$  appears to be particularly reasonable intuitively.

This belief function takes the general form that any contract offer  $C_2 = \{i_2, i_2, \rho\}$ , where  $\rho$  is very low, is interpreted by  $P2$  as signaling an overly redistributive contract  $C_1$ , and is consequently rejected by  $P2$ . On the other hand, all contract offers  $C_2 = \{i_2, i_2, \rho\}$ , with  $\rho$  in excess of  $\rho^*$  inspire sufficient confidence to  $P2$  that  $P1$  and  $A$  have written an *efficient contract*  $C_1^{fb}$ , that  $P2$  accepts to lend under the terms of the contract offer  $C_2$ .

The cut-off  $\rho^*$  in the belief function  $\nu_2(C_1 | C_2)$ , that supports the Bayes-Nash equilibrium is determined as follows. A contract offer  $C_2 = \{i_2, i_2, \rho\}$  allows  $P2$  to fully understand the contract  $C_1$  with probability  $P(\rho) = \frac{\rho}{\rho + \kappa}$ . Thus,  $P2$  would certainly reject  $C_2$  at least with probability  $P(\rho)$  if contract  $C_1$  were too redistributive and were to contain a negative covenant  $\Phi_1$  that is too constraining. Even if  $P2$  were expected to accept the redistributive contract with probability  $(1 - P(\rho))$ , the most  $P1$  and  $A$  can then hope to get in the good state with a maximally redistributive contract  $C_1$  followed by a contract offer  $C_2 = \{i_2, i_2, \rho\}$  is:

$$\left(\frac{\rho}{\rho + \kappa}\right)L + \left(1 - \frac{\rho}{\rho + \kappa}\right)V_x.$$

<sup>38</sup> In contrast, if  $P1$  and  $A$  were to agree on the *efficient contract*  $C_1^{fb}$ , followed by the same contract offer  $C_2 = \{i_2, i_2, \rho\}$  they would get:

$$X - i_2 - c,$$

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<sup>38</sup>The maximally redistributive contract  $C_1$  would set  $\Phi_1 = 0$ , so that  $P1$  and  $A$  would be able to claim the entire cash-flow net of effort costs:  $(X - c)$ .

In principle, the law could even allow for negative  $\Phi_1$ , implying that  $P1$  could seize  $P2$ 's property (over and above  $i_2$ ) if  $P2$  makes a loan. In a world with no reading costs, there would be no loss in enforcing these extremely redistributive contracts, because  $P2$  would never sign them.

since then  $P2$  would accept to invest in the firm with probability 1. Thus, when  $A$  is expected to make a contract offer  $C_2 = \{i_2, i_2, \rho\}$  with due diligence  $\rho$  in the good state such that

$$X - i_2 - c \geq \left(\frac{\rho}{\rho + \kappa}\right)L + \left(1 - \frac{\rho}{\rho + \kappa}\right)V_x \quad (8)$$

the best response for  $P1$  and  $A$  is to settle on the *efficient contract*  $C_1^{fb}$  at date 1. Therefore, an updated belief for  $P2$  such that  $\nu_2(C_1^{fb} | C_2 = \{i_2, i_2, \rho\}) = 1$  for any contract offer  $C_2 = \{i_2, i_2, \rho\}$  where  $\rho$  satisfies (8), may be consistent with equilibrium play. Similarly, an updated belief  $\nu_2(C_1^{fb} | C_2 = \{i_2, i_2, \rho\}) = 0$  for a contract offer  $C_2$  where  $\rho$  that *does not* satisfy (8) is consistent with equilibrium play. Therefore, the cut-off  $\rho^*$  is given by the solution  $\rho$  for which (8) holds as an equality:

$$\rho^* = \frac{\kappa\{V_x - (X - i_2 - c)\}}{X - i_2 - c - L}. \quad (9)$$

The belief-function for  $P2$  described above supports what is often referred to as *the least-cost separating equilibrium* of our contracting game. That is, under this belief function  $P1$  and  $A$  are able to credibly signal at the lowest possible cost, in terms of (wasteful) due diligence expenditure  $\rho^*$ , that they agreed to the efficient contract  $C_1^{fb}$  at date 1.

For the remainder of our analysis we shall focus on this *least-cost separating equilibrium* and make the following slightly modified assumptions to reflect the presence of positive reading costs:

Under these assumptions, and under the belief-function  $\nu_2(C_1^{fb} | C_2 = \{i_2, i_2, \rho\}) = 1$  if  $\rho \geq \rho^*$  and  $\nu_2(C_1^{fb} | C_2 = \{i_2, i_2, \rho\}) = 0$  if  $\rho < \rho^*$ , the *least-cost separating* Bayes-Nash equilibrium of the full contracting game takes the following form:

**Proposition 7** *Under assumptions (A1, A2, A3b, A4), the least cost separating Bayes-Nash equilibrium of the lending game with reading costs is as follows:*

*At date 1, P1 and A agree on contract  $C_1^{fb}$  taking the following form:*

- 1. P1 lends  $i_1 + \rho^*$  to A. In turn, A invests  $i_1$  in the project and holds  $\rho^*$  until date 2;*
- 2. P1 obtains a senior debt claim of  $F_1 = \frac{i_1 + \rho^* - (1 - \pi)(L + \rho^*)}{\pi}$ , and a commitment not to pledge more than  $\Phi_1 = i_2$  to P2;*

*At date 2:*

1. *A offers contract  $C_2 = \{i_2, i_2, \rho^*\}$  to P2 in the good state,*
2. *P2 accepts the contract, undertakes the due diligence and invests  $i_2$  in the firm,*
3. *The agent A then chooses high effort ( $e = 1$ ) and the project yields  $X$  at date 3;*
4. *In the bad state of nature P2 refuses to lend and the project is liquidated, paying  $L + \rho^*$  to P1.*

**Proof.** Given that under due diligence  $\rho^*$  we have

$$X - i_2 - c = \left( \frac{\rho^*}{\rho^* + \kappa} \right) L + \left( 1 - \frac{\rho^*}{\rho^* + \kappa} \right) (X - c), \quad (10)$$

it is a (weak) best response for P1 and A to agree to contract  $C_1^{fb}$ . Thus, P2's equilibrium beliefs  $\nu_2(C_1^{fb} \mid C_2 = \{i_2, i_2, \rho^*\}) = 1$  are consistent with P1 and A's equilibrium play, and it is a (weak) best response for A to offer contract  $C_2 = \{i_2, i_2, \rho^*\}$  at date 2, and a (weak) best response for P2 to accept  $C_2$  in the good state, but to reject it in the bad state. In particular, A cannot obtain a higher payoff by offering any other contract  $C_2 = \{i_2, i_2, \rho\}$ , with  $\rho \neq \rho^*$  at date 2. Indeed, any contract with  $\rho > \rho^*$  would involve unnecessarily high due diligence expenditures, and any contract such that  $\rho < \rho^*$  would be rejected by P2 given his updated beliefs  $\nu_2(C_1^{fb} \mid C_2 = \{i_2, i_2, \rho\}) = 0$  and therefore would only yield a payoff of  $L + \rho^*$  to P1 and A. To show this is less than  $X - i_2 - c$ , note that by A3b,  $\pi(R_g - \rho^*) + (1 - \pi)L \geq i_1$ . Combining this assumption with  $L < i_1$ , and the definition of  $R_g$ , it follows that  $X - i_2 - c > R_g > L + \rho^*$ . ■