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Macroeconomic Theory for a World of Imperfect Knowledge

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Roman Frydman and Michael D. Goldberg

Abstract

We have recently proposed an alternative approach to economic analysis, which we call Imperfect Knowledge Economics (IKE). Although IKE builds on the methodology of contemporary macroeconomics by modeling aggregate outcomes on the basis of mathematical representations of individual decision making, it jettisons models that generate sharp predictions. In this paper, we elaborate on and extend the arguments that led us to propose IKE. We show analytically that in order to avoid the fundamental epistemological flaws inherent in extant models, economists must stop short of fully prespecifying change. We also show how acknowledging the limits of their knowledge may enable economists to shed new light on the basic features of observed time-series of market outcomes, such as fluctuations and risk in asset markets, which have confounded extant approaches for decades.

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Part I

From Early Modern Economics To Imperfect Knowledge Economics

Early on as graduate students, we were introduced to the Rational Expectations Hypothesis (REH)—a radical solution to the recalcitrant problem of modeling individuals' forecasting behavior. Having read Hayek and Keynes, both of us had our doubts about this facile attempt to represent how individuals forecast the future with a single model written down by an economist. As our own research developed, we came to understand that the RE movement that had swept the profession in the 1970's was in fact a mechanical response to Professor Phelps's earlier revolution in macroeconomics.

Phelps set out to model aggregate outcomes on the basis of genuine representations of individual behavior. He accorded an autonomous role to market participants' expectations. To capture the idea that expectations matter for market outcomes and yet individuals do not form the same forecast, Phelps formulated his well known "island model." The island parable brought into sharp focus the fact that market participants have to cope with being ignorant of the future—and even much of the present. In Phelps's vivid depiction, "isolated and apprehensive, these Pinteresque figures construct expectations of the state of the economy...and maximize relative to that imagined world."

Surprisingly, the RE counterrevolution presumed that individuals' "imagined world" could be captured with mechanical rules. Phelps understood early on that REH was fundamentally flawed, and that it derailed his program to develop genuine microfoundations for macroeconomics. Unfortunately, the illusion of modeling precisely how the market thinks about the future was just too seductive. Nowadays, all graduate students of economics—and, increasingly, undergraduates, too—are taught that to capture rational, self-interested behavior in a "scientific" way, they must use REH.

Phelps's early critique of REH gave us the courage to be similarly critical. Many ideas that he shared with us over several decades helped shape our work on an alternative that would be more faithful to his genuine micro-foundations approach. Imperfect Knowledge Economics (IKE) is our attempt at such an alternative.

1 Modern Macroeconomics: Individual Forecasting and Aggregate Outcomes

When macroeconomists confront their models with time-series data they often find gross inconsistencies. In our recent book (Frydman and Goldberg, 2007), we argue that contemporary models also suffer from insuperable epistemological flaws. We trace these empirical and theoretical difficulties to a common source: in modeling aggregate outcomes, contemporary economists fully prespecify the causal mechanism that underpins change in real-world markets. They do so because they have come to believe that, in order to be worthy of scientific status, their models should generate sharp predictions of how individual decision making and market outcomes evolve over time.¹

Although IKE builds on the methodology of contemporary macroeconomics by modeling aggregate outcomes on the basis of mathematical representations of individual decision making, it jettisons models that generate sharp predictions. In this paper, we elaborate on and extend the arguments that led us to propose IKE. We show analytically that in order to avoid the fundamental epistemological flaws inherent in extant models, economists must stop short of fully prespecifying change. We also show how acknowledging the limits of their knowledge may enable economists to shed new light on the basic features of observed time-series of market outcomes, such as fluctuations and risk in asset markets, which have confounded extant approaches for decades.

Modern macroeconomics constructs models of aggregate outcomes on the basis of mathematical representations of individual decision making, with market participants' forecasting behavior lying at the heart of the interaction between the two levels of analysis. Individuals' forecasts play a key role in how they make decisions, and markets aggregate those decisions into prices. The causal mechanisms behind both individual decisions and aggregate outcomes, therefore, depend on market participants' understanding of the economy and how they use this knowledge to forecast the future.

By focusing on the central role of forecasting for understanding the connection between micro and macro outcomes, economists have achieved important insights. For example, building on the path-breaking work of Phelps (1968, 1970), Lucas (1976) sharply criticized econometric policy analysis that examined the effects of changes in tax rates, money supply, or other "policy" variables on market outcomes using Keynesian aggregate models. This

¹As we illustrate rigorously in section 7, a contemporary model is said to generate sharp predictions of change if, conditional on the probability distribution of outcomes at some initial time, it represents outcomes at any other time, past or future, with a single probability distribution.

analysis presumed that the same structure—the set of causal variables and the parameters that relate them to those variables—would continue to represent adequately the causal mechanism after a change in policy. The main point of the “Lucas critique” was the untenability of that premise. He argued that changes in policy variables would alter the way market participants forecast the future—and hence their decision-making. In general, this change on the individual level would also alter the causal mechanism driving market outcomes.

Lucas offered a seemingly straightforward remedy to this fundamental difficulty. He presumed that the Rational Expectations Hypothesis (REH) would enable economists to model exactly how policy changes would affect market participants’ forecasts and aggregate outcomes. REH postulates that the economist’s aggregate model represents the precise causal mechanism driving individuals’ forecasts and their revisions. By constraining the predictions on the individual and aggregate levels to be one and the same, REH is thought to offer a “scientific” way to predict both the micro and macro effects of policy changes. In embracing REH-based models, economists have merely replaced Keynesian policy analysis with another mechanistic approach to evaluating the consequences of policy changes.

To be sure, the Lucas critique of Keynesian models does not depend on REH: it requires only that policy changes influence forecasting strategies significantly enough to alter the causal mechanism driving market outcomes. What has been largely overlooked, however, is that Lucas’s critical arguments point to a fundamental difficulty inherent in the entire modern research program in macroeconomics. After all, while policy changes undoubtedly play a role in market participants’ alteration of their forecasting strategies, so do many other factors.²

In fact, even if one were to limit the analysis solely to policy changes, the solution that Lucas proposes is facile: REH supposes that individuals revise their forecasting strategies in mechanical ways that can be precisely specified in advance. However, in capitalist economies, individuals are strongly motivated to search for genuinely new ways to forecast the future and deploy their resources. The social context, including the institutions within which individuals make decisions, also changes in unforeseeable ways. But when the social context or how market participants forecast future outcomes changes, so too does the causal mechanism underpinning market outcomes. Thus, change in

²For some early warnings concerning the fundamental flaws of the REH approach, see Frydman (1982), Phelps (1983), and Frydman and Phelps (1983).

capitalist economies is to a significant extent non-routine, for it does not follow pre-existing rules and procedures. The premise of IKE is that leaving macroeconomic models open to such change is crucial for understanding outcomes in real-world markets.

2 Early Modern Narrative Accounts: Respecting the Limits to Knowledge

The history of economic thought includes widely differing responses to the daunting challenge that change poses for economic analysis. Early modern economists relied on a largely narrative mode of analysis. Although imprecise by contemporary standards, narrative accounts had the important advantage of leaving economists relatively free to explore the complexity and opaqueness of the interdependence between individual rationality, the social context of decision-making, and market outcomes.³ Indeed, the giants of early modern economics uncovered remarkably powerful and durable insights, such as Hayek's (1948) prescient prediction that socialist planning is bound *in principle* to fail; Knight's (1921) assertion that standard probabilistic uncertainty cannot adequately characterize business decisions; and Keynes's (1936) closely related arguments concerning the importance of radical uncertainty, the social context, and conventions for forecasting returns and risk on investment in real and financial assets. These insights point to the fundamental flaw in contemporary economists' research program: the causal mechanism that underpins change in capitalist economies is not completely intelligible to *anyone*, including market participants, economists, policy officials, or social planners.

3 Contemporary Models: Fully Prespecifying Change

Largely ignoring early modern arguments concerning the inherent limits to economists' knowledge, contemporary economists construct models that we call *fully predetermined*. These models represent the causal mechanism that underpins change on the individual and aggregate levels with mechanical rules. Thus, they leave no room for changes in individual decision making and aggregate outcomes that have not been fully specified in advance by an economist.

Economists represent an individual's decision making by specifying her forecasts of future market outcomes, preferences that rank the future consequences of her decisions for her well being, and the constraints that she faces.

³Of course, a narrative mode of analysis also constrains argument, but this constraint is relatively weak compared to the rigor of mathematical language.

They also specify a decision rule, such as maximization of an individual's well being, which selects the preferred deployment of resources. In this way, economists represent the way an individual makes decisions in terms of a set of causal factors.

Alternative specifications of preferences, forecasting strategies, constraints, and decision rules enable economists to formalize competing explanations of individual decision making.⁴ At some arbitrary "initial" point in time, these representations relate an individual's choices to causal factors with qualitative conditions.⁵ However, although these representations of individual decision-making are qualitative, contemporary economists fully prespecify change by imposing restrictions that relate their representations at all points in time, past and future, *exactly* to the properties of their representation at the initial point in time. Based on such microfoundations, contemporary models also fully prespecify, in terms of some set of causal factors, how market outcomes unfold over time.

As time passes, individuals alter the way they to make decisions, at least intermittently. In general, therefore, models with structures that vary over time are needed to represent adequately individual decision making. Oddly, in view of the broad applicability of the Lucas critique, the vast majority of contemporary models do not allow for any change in their structure. These time-invariant models do not accord any role to revisions of forecasting strategies in driving outcomes; instead, they force economists to rely solely on the movement of causal factors to explain time-series data. These factors are represented as random variables that depend on "exogenous shocks." The random variation in these shocks is supposed to account for new information that becomes available to market participants. Consequently, contemporary economists focus on information and its asymmetries as the principal factors driving market outcomes. As with the rest of the structure of contemporary models, the probability distributions of the causal variables or new information are usually assumed to be time-invariant.

Contemporary economists sometimes recognize the importance of incorporating in their models the fact that participants in real-world markets do not adhere endlessly to one forecasting strategy, or, more broadly, that they at times alter the way they make decisions. They also sometimes take into account the fact that the social context, particularly economic policy, can change

⁴For an overview of alternative specifications of these components and how they result in alternative representations of individual decision-making, see chapter 3 of our book.

⁵For example, it is common for economists to assume that an individual's utility depends positively on her consumption of goods or that her forecast of a future market price depends positively on the current value of this price.

over time. However, because their models determine exactly how individual decisions and the social context unfold over time, they remain as mechanistic as their time-invariant counterparts.⁶ By adhering to the bogus belief that only models that promise sharp predictions deserve scientific status, contemporary economic analysis locks in the presumption that individuals never forecast or alter their decision-making in new ways.

Of course, economists recognize that knowledge, including their own, is imperfect and that time-invariant models or those that fully prespecify change are fraught with error. To represent their own imperfection of knowledge, they include additive error terms to their models. As with new information, economists usually assume that the distributions characterizing these error terms never change, or if change is allowed, they fully prespecify it.⁷ Here, again, contemporary economics embodies an odd and counterproductive conception of imperfect knowledge. By fully predetermining how the probability distributions of the error terms in his model evolve over time, an economist, in effect, fully prespecifies how his own imperfection of knowledge unfolds between the initial time and all other time periods.

3.1 Rational Expectations Models

Contemporary economists have adopted a set of *a priori* assumptions that putatively characterize how rational individuals make decisions, with REH as the centerpiece of their standard of rationality. In REH models, people's beliefs "are not inputs," but the outcomes of economists' theories.⁸ Thus, REH rules out the possibility that market participants' forecasting strategies play an autonomous role in driving individual decision making and aggregate outcomes. An REH model derives its representation of forecasting strategies from its specification of preferences, constraints, and the way that policy and other causal variables unfold over time. As such, the causal variables and parameters that make up an REH representation of forecasting behavior stem solely from variables and parameters that an economist uses to specify the other components of his model.

⁶A popular way to model policy changes in the literature is to use the Markov-switching framework of Hamilton (1988, 1990). These models fully prespecify the set of possible structures that might represent outcomes and a probabilistic rule that determines the timing of change. Consequently, as we show in Frydman and Goldberg (2007, chapter 6), the representations of aggregate outcomes implied by Markov-switching models are formally equivalent to those produced by models whose structure is time-invariant.

⁷For a well-known formulation that fully prespecifies stochastic terms within an economist's model, see Engel (2003).

⁸Thomas J. Sargent, in an interview with Evans and Honkapohja (2005, p. 566).

Proponents of REH-based models regard this “tightness” as the greatest virtue of their approach. They often point to the fact that it “disciplines” economic analysis in a way that was absent in earlier approaches. Indeed, to inculcate and enforce this discipline in the economics profession, every graduate student is warned early on to “[b]eware of theorists bearing free parameters [arising from autonomous representations of forecasting strategies].”⁹ Following this dictum, the vast majority of economists, whom we refer to as *conventional*, appeal to REH in specifying the microfoundations of their models.

Unsurprisingly, conventional models have experienced the most glaring empirical failures in financial and other markets in which revisions of forecasting strategies are among the key factors driving outcomes. Many of these failures were uncovered by successive generations of conventional economists themselves.¹⁰ Barred from constructing models that accord an autonomous role to market participants’ forecasts, REH theorists have engaged in an intensive effort to explain outcomes on the basis of alternative specifications of the other components of their models. For example, in attempting to explain the excess return of stocks over bonds as a risk premium, REH has led economists to search for alternative specifications of preferences.¹¹

3.2 Behavioral Models

Behavioral economists, for their part, have also uncovered many inconsistencies between the way market participants “actually” behave and standard representations of rational behavior. However, they have not interpreted their findings as evidence that the contemporary standard of rationality does not adequately represent rational decision-making. Instead, they have concluded that market participants are not “fully rational.” This view has led some behavioral economists to use parts of the contemporary standard of rationality to specify certain components of their models, while replacing others with their empirically motivated alternatives.

⁹ Attributed to Robert E. Lucas, Jr. in Sargent (2001, p.73).

¹⁰ Frydman and Goldberg (2007, chapters 7 and 8) provide an extensive discussion of the empirical failures of REH models in the context of currency markets, along with more than 100 references to research documenting these failures. The list of such references would grow considerably if the conventional approach’s failures in other markets were included.

¹¹ This research strategy has an important drawback. As Lucas (2003) points out, preference specifications designed to explain outcomes in one market are often inapplicable or not useful in modeling other markets. This is not surprising because representations of changes in preferences in REH models must capture the effect of changes in market participants’ preferences and knowledge, and the latter clearly depend on the modeling context.

For example, some behavioral-finance theorists have continued to rely on REH to represent individual forecasting. Thus, they follow their conventional colleagues in searching for alternative specifications of preferences to remedy the failure of canonical REH models.¹² However, the behavioral approach has an important methodological advantage over its conventional counterpart: it recognizes that forecasting plays an autonomous role in driving markets and thus admits models that do not rely on REH. As a result, behavioral economists have looked for empirically motivated alternatives to REH. This search has yielded a number of valuable insights on how individuals form and revise their forecasts. For example, they have documented a regularity called “conservatism:” individuals tend to revise the strategies that they use to form beliefs about uncertain outcomes in ways that lead to gradual changes in those beliefs.¹³

Admitting such departures from REH into the microfoundations of economic models is an important advance in macroeconomics. However, behavioral economists have followed their conventional colleagues in insisting that, to be worthy of scientific status, their models should generate sharp predictions. Consequently, they have formalized their behavioral insights into how individuals revise their beliefs with mechanical rules that specify exactly all change in advance.

3.3 Internal Inconsistency and Flawed Microfoundations

Beyond ignoring key factors that drive market outcomes, the practice of fully prespecifying change creates insuperable epistemological problems for modern macroeconomics. These problems stem from the standard use of probability theory to represent decision-making under uncertainty.

3.3.1 Standard Probabilistic Representations

Once an economist characterizes the causal factors of his model as random variables, his model becomes probabilistic. Such models represent the causal

¹²For a widely-cited behavioral model of the risk premium that relies on REH, see Barberis et al. (2001). This study, which augments risk averse preferences with the assumption of loss aversion, reports the results of calibration exercises that are supportive of the model in the market for equities. However, Frydman and Goldberg (2007, chapter 13) show that, although the Barberis *et al.* model might appear successful according to the calibration methodology in the market for equities, it is grossly inconsistent with the observed time path of currency prices.

¹³See Edwards (1968) and Shleifer (2000). We make use of this regularity in modeling long swings in asset prices. See section 12.1.

mechanism driving outcomes on the individual level at a point in time with a probability distribution that is conditional on the set of causal variables. A time-invariant model represents outcomes at every point in time with the same conditional distribution. However, even if a contemporary economist allows for change, he fully prespecifies when and how the conditional probability distributions implied by his model vary over time. Because all transitions across probability distributions are fully predetermined, such models in effect characterize individual decision making at every point in time with the same overarching probability distribution.¹⁴

3.3.2 Diversity and Gross Irrationality

Most contemporary economists ignore the diversity of forecasting strategies that we observe in real world markets. To represent such diversity, an economist would need to specify more than one conditional probability distribution on the individual level. However, if he were to do so, at least some, if not all, of those distributions would systematically differ from the single overarching probability distribution—a sharp prediction—generated by the aggregate model. Thus, any fully predetermined model that recognizes diversity in how market participants forecast the future is *necessarily* internally inconsistent.

Lucas (1995, pp. 254-255; 2001, p.13) has forcefully argued against such models. An economist hypothesizes that his model adequately represents regularities in market outcomes; market participants should be viewed as irrational if their forecasting strategies were systematically inconsistent with those regularities. Indeed, in his Nobel lecture, Lucas (1995, p. 255) pointed to internal inconsistency as the key difficulty in constructing macroeconomic models based on representations of individual decision making.

The prevailing strategy for macroeconomic modeling in the early 1960's held that the individual or sectoral models arising out of this intertemporal theorizing could then simply be combined in a single model. But models of individual decisions over time necessarily involve expected future prices. . . . However, . . . [aggregate] models assembled from such individual components implied behavior of actual prices. . . that bore no relation to, and were in general *grossly inconsistent* with, the price expectations that the theory imputed to individual agents (Lucas, 1995, pp. 254-255, emphasis added).

¹⁴For a rigorous demonstration, see chapter 6 of our book. See also section 7 below.

Internally inconsistent models presume that market participants adhere to strategies that generate systematic forecast errors, and thus attribute to them gross irrationality. For Lucas, therefore, any such model of time-series regularities is “the wrong theory.”

Lucas’ argument is compelling. However, avoiding inconsistency between a model’s representations on the individual and aggregate levels is not as simple as it might appear: in a world of imperfect knowledge, economists must jettison sharp predictions—the *sine qua non* of Lucas’s own approach.

3.3.3 REH

Otherwise, an economist is left with only one way to rid macroeconomic models of internal inconsistency: he must represent market participants’ forecasting strategies with the one probability distribution generated by the aggregate model that he himself constructs. Indeed, the promise of internal consistency is precisely why Lucas and others embraced REH. As Lucas later put it, “John Muth’s [REH] focused on this inconsistency... and showed how it can be removed” (Lucas, 1995).

Although REH automatically removes internal inconsistency from an economist’s model, Muth understood that it should *not* be viewed as a normative hypothesis about how rational individuals should forecast the future. As he put it,

At the risk of confusing this *purely descriptive* hypothesis with a pronouncement as to what firms ought to do, we call such expectations “rational” (Muth, 1961, p. 316, emphasis added).

Of course, what applies to firms is true of other market participants as well. Unfortunately, despite Muth’s warning, REH is, in fact, commonly interpreted as a pronouncement as to what they ought to do.

Early critics pointed out a number of reasons why REH should not be relied on to represent adequately market participants’ forecasting strategies, let alone how rational—purposeful—individuals forecast the future.¹⁵ Frydman (1982) argued that there is an inherent conflict between REH’s presumption that “people’s beliefs” can be adequately represented with the prediction of an economist’s model and the premise that market participants are motivated by self-interest: purposeful individuals would not, in general, adhere to

¹⁵Later critics built on these arguments. Particularly striking are the arguments by Sargent (1993), who was an early advocate of REH. For further discussion and references, see chapter 2 of our book.

a single forecasting strategy, let alone the strategy implied by an economist's model. Moreover, as Phelps (1983) pointed out, economists themselves have constructed a number of alternative models of outcomes. Thus, if a particular economist's model were somehow to represent rational forecasting, the use of REH in any other model to represent forecasting would have to presume gross irrationality. As we argue below, the only way to escape this conundrum is to jettison the belief that rational decision-making can be fully prespecified by an economist.

REH and the Mathematics of Planning REH models avoid the foregoing difficulties by ignoring diversity among economists' models and market participants' forecasting strategies. Although they sometimes allow for differences in preferences and information,¹⁶ they represent forecasts with a single overarching probability distribution. Such representations are usually referred to as the "representative agent's" forecasting strategy.

While all economic models are abstractions, the representative-agent assumption is particularly extreme: it ignores the division of knowledge, which is the key feature that distinguishes the allocation of resources by decentralized markets from an "optimal" deployment of resources by a single individual. As Hayek put it,

The economic problem of society is...not merely a problem of how to allocate "given" resources – if "given" is taken to mean given to a single mind which deliberately solves the [resource-allocation] problem....It is rather a problem of how to secure the best use of resources known to any of the members of society, for ends whose relative importance only these individuals know. Or, to put it briefly, it is *a problem of the utilization of knowledge which is not given to anyone in its totality* (Hayek, 1945, p. 519-520, emphasis added).

In relying on REH, economists have ignored Hayek's arguments. Indeed, Lucas's account of how REH led him to embrace the representative-agent construct stands in stark contrast to Hayek's position. In discussing market outcomes for a competitive industry under perfect foresight, which is the deterministic analog of REH, Lucas (2001, p. 13) pointed out that "one can show that an industry over time will operate so as to maximize a discounted,

¹⁶See Lucas (1973) for an early example and Stiglitz (2001) for an overview of informationally driven explanations of market outcomes.

consumer surplus integral—a problem that is mathematically no harder than the present value maximizing problem faced by a single firm.”

Lucas then asked “who, exactly, is solving this planning problem?” As Hayek did, he recognized that “Adam Smith’s ‘invisible hand,’ of course, *not any actual person*” (emphasis added). Nevertheless, in a striking leap of faith, Lucas claimed that an economist—an actual person—can adequately represent what the invisible hand of the market does by solving the value-maximizing problem faced by a single firm.

For Hayek, the division of “knowledge which is not given to anyone in its totality” was the key to his argument that central planners could not, *in principle*, substitute for markets. For Lucas, REH models, which rule out the division of knowledge and enable an economist to make use of single-agent optimization techniques, were the right tools to comprehend market outcomes.¹⁷ As he put it,

[T]he mathematics of planning problems turned out to be just the right equipment needed to understand the decentralized interactions of a large number of producers. (Lucas, 2001, p.14)

In effect, Lucas posited that Smith’s invisible hand could be made visible and intelligible, after all. To understand markets, economists need only learn how to solve optimal allocation problems that a fictitious central planner confronts. Indeed, this is what graduate students in economics are instructed to devote most of their time to doing.

Gross Irrationality of REH Representations Contemporary economists use the “mathematics of planning” to model market outcomes partly because they believe that the representative-agent assumption is just a harmless approximation. However, our foregoing arguments make clear that this abstraction is in fact a knife-edge assumption. Once one recognizes the smallest degree of diversity of forecasting strategies, REH models become internally inconsistent: even if, somehow, an REH model were to represent adequately the average of those strategies, it would presume *ipso facto* that an

¹⁷As we pointed out above, Lucas (1973), Stiglitz (2001), and others have relied on imperfect and asymmetric information in REH models to represent heterogeneity of forecasts in decentralized markets. Frydman (1983) formalizes the arguments in the so-called “Socialist Calculation Debate” by von Mises (1920) and Hayek (1945). Frydman shows that because REH models necessarily ignore the imperfection of knowledge concerning the causal mechanism, Von Mises’ and Hayek’s arguments against planning apply wholesale to contemporary REH models that attempt to capture “the decentralized interactions of a large number of producers” solely with informational imperfections.

individual adhered to a forecasting strategy that generates systematic forecast errors *endlessly*. Thus, REH models presume that market participants, who do hold diverse forecasting strategies, are grossly irrational. By Lucas' own argument, they are the wrong theory of market outcomes.

3.3.4 Behavioral Models

The microfoundations of behavioral models often rely on the representative-individual construct. Sometimes, however, behavioral economists attempt to capture the fact that participants in real-world markets employ different forecasting strategies. But, even if they allow for diversity, behavioral economists, too, embrace the conventional belief that economic models should generate sharp predictions and thus fully prespecify change on the individual and aggregate levels.

For an approach that aims for psychological realism, representing market participants as robots who act according to rules that are fully prespecified by an economist is odd. But what makes this approach's reliance on fully predetermined models particularly puzzling is that it was developed after the REH revolution, the rationale for which was that fully predetermined non-REH models, including non-REH behavioral representations, involve an inherent inconsistency between their probabilistic implications on the individual and aggregate levels. As Lucas argued, internally inconsistent models are the wrong theory of time-series regularities.

3.3.5 Sharp Predictions and the Modern Research Program

The contemporary methodology raises an intractable epistemological problem: there is an inherent conflict between the objective of modeling market outcomes on the basis of explicit, plausible microfoundations and the insistence of both conventional and behavioral economists that their models generate sharp predictions. In real world markets, contemporary models are internally inconsistent, and their "microfoundations" represent grossly irrational behavior.

4 The Promise of Imperfect Knowledge Economics

We have traced the empirical failures and epistemological flaws of contemporary models to economists' insistence on sharp predictions. In part II, we use

a simple algebraic model of a market price to show that in order to escape these flaws, economists must abandon this position. Part III uses the same simple model to illustrate how we have used IKE to model fluctuations and risk in asset markets—phenomena that have confounded conventional models for decades. It also shows how IKE avoids the epistemological flaws of extant approaches.

4.1 Qualitative Models of Change

IKE continues the modern research program in macroeconomics, which was interrupted by the REH revolution. The giants of early modern economics (Knight, 1921; Keynes, 1921, 1936; Hayek, 1945, 1948) and the originators of the modern micro-based approach to macroeconomics (Phelps, 1968, 1970) emphasized the importance of forecasting for understanding market outcomes. However, they also argued that the key feature of capitalist market economies is that they engender change that cannot be prespecified with mechanical rules. At the time Phelps pioneered modern macroeconomics, it was not apparent how to leave mathematical models open to autonomous, non-fully prespecified revisions of forecasting strategies while still representing individual decision-making mathematically. We propose IKE as such an approach and compare it with extant approaches to modern macroeconomics.

Like contemporary models, IKE models consist of representations of an individual's preferences, the constraints that she faces, her forecasts of the future outcomes that are relevant to her wellbeing, and a decision rule that selects her preferred deployment of resources. However, IKE recognizes that knowledge is inherently imperfect: no one has access to a fully predetermined model that adequately represents, as judged by whatever criteria one chooses, the causal mechanism that underpins outcomes in all time periods, past and future. Consequently, IKE does not fully prespecify which causal variables may be relevant, or when and how these variables may enter an economist's representation of forecasting behavior. In this way, IKE models remain open to changes in the ways individuals in real-world markets forecast the future—ways that they themselves, let alone economists, cannot specify in advance.

Although IKE jettisons sharp predictions, it aims to explain aggregate outcomes on the basis of mathematical representations of individual decision making. To this end, IKE explores the possibility that revisions of forecasting strategies, though diverse and context-dependent, might exhibit qualitative regularities that can be formalized with mathematical conditions. An aggregate model based on such microfoundations generates only qualitative predictions of market outcomes.

4.1.1 Non-Standard Use of Probabilistic Formalism

Contemporary models represent outcomes at each point in time – and thus how they unfold over time – with a single “overarching” conditional probability distribution.¹⁸ The relationships between the moments of this distribution and the set of causal variables constitute the model’s empirical content that can be confronted with the time-series data.

By contrast, early modern economists argued that standard probabilistic representations cannot adequately represent change. Indeed, both Knight and Keynes emphasized that economic decisions and institutional and policy changes are fraught with *radical uncertainty*; the complete set of outcomes and their associated probabilities can neither be inferred from past data nor known in advance.

Radical uncertainty is often thought of as a situation in which no economic theory is possible: neither economists (nor market participants) are able to represent mathematically any aspects of the causal mechanism underpinning change. IKE adopts an intermediate position between radical uncertainty and the contemporary presumption that models that fully prespecify change are not only within reach of economic analysis, but anything less is not worthy of scientific status.

Of course, if economic decisions stem only from erratic “animal spirits,” no economic theory is possible. As Phelps (2008) has recently put it, “animal spirits can’t be modelled.” Although animal spirits may play a role, IKE explores the possibility that individual decision-making displays some qualitative regularity that can be represented with a mathematical model.

Departing from the position of Knight and Keynes, IKE makes use of the probabilistic formalism. This facilitates the formalization of conditions that specify the microfoundations of IKE models and the mathematical derivation of their qualitative implications. However, IKE recognizes the importance of early modern arguments that market participants, let alone economists, have access to only imperfect knowledge of which causal factors may be useful for understanding outcomes and how they influence those outcomes.

Like extant approaches, IKE represents revisions of market participants’ forecasting strategies, and more broadly change in how individuals make decisions, with transitions across probability distributions. However, IKE constrains these revisions with only qualitative conditions. Consequently, it does not follow extant approaches in presuming that individual decision making and market outcomes can be adequately represented with a single overarching probability distribution. At the same time, IKE does not adopt the other

¹⁸See chapter 6 of our book and section 7 below.

extreme position that uncertainty is so radical as to preclude economists from saying anything useful and empirically relevant about how market outcomes unfold over time.

Because its restrictions on change are qualitative, IKE models represent outcomes at every point in time with myriad probability distributions. Nevertheless, the qualitative restrictions of IKE models constrain all transitions across probability distributions to share one or more qualitative features. These common features, which are embodied in what we call *partially predetermined probability distributions*, enable economists to model mathematically some aspects of the causal mechanism that underpin individual decision making and market outcomes.¹⁹ Such probabilistic representations constitute the empirical content of IKE models.

Although IKE acknowledges the limits to knowledge, it constrains its models sufficiently to distinguish empirically among alternative explanations of aggregate outcomes. In our book, we develop several alternative IKE models and show that their qualitative predictions enable us to reject some in favor of others on the basis of time-series data. Jettisoning sharp predictions may appear to lower the “scientific standard” that economists have self-imposed on their models. However, as Hayek anticipated, replacing the “pretense of exact knowledge” with imperfect knowledge as the foundation for economic analysis is crucial for understanding markets. Remarkably, stopping short of sharp predictions is also necessary to escape the epistemological flaws of extant fully predetermined models.

4.2 Avoiding Epistemological Flaws of Contemporary Models

As we mentioned above, although some well-known behavioral models use REH, the behavioral approach admits non-REH representations of forecasting strategies, thereby according forecasts an autonomous role in driving market outcomes.

The acceptance of the behavioral approach has weakened the position of REH as *the* way to model forecasting behavior. But, it has not diminished contemporary economists’ insistence on models that generate sharp predictions. Indeed, despite their focus on psychological realism, behavioral economists fully prespecify their non-REH representations, and leading behavioral economists have emphasized that theirs “is not meant to be a separate approach [of

¹⁹For some simple examples of partially predetermined probability distributions, see chapter 3 of our book.

contemporary economics] in the long run” (Camerer, Loewenstein and Rabin, 2004, p. 42).

Like the behavioral approach, IKE accords market participants’ forecasting strategies an autonomous role in driving individual decision-making and hence market outcomes. However, because it represents revisions of these strategies with qualitative conditions, IKE enables economists to avoid the internal inconsistency inherent in behavioral models. The key reason for this assertion is already apparent in our foregoing discussion of fully predetermined models.

On the aggregate level, the predictions of an IKE model are characterized with myriad distributions, whereas on the individual level, a pluralism of distributions is required to represent diversity of forecasting strategies. Thus, by limiting itself to qualitative predictions on the aggregate level, IKE opens the microfoundations of its models to a diversity of strategies without necessarily introducing internal inconsistency.

Although jettisoning sharp predictions is necessary to avoid internal inconsistency, it is not sufficient. As we discuss more fully in section 14, an IKE model avoids inconsistency in different ways, depending on the predictions that it generates at the aggregate level and which aspects of the causal mechanism it constrains on the individual level. We show that an IKE model can avoid internal inconsistency, and thus the presumption of irrationality, even if it assumes that the diversity of forecasting strategies includes some that predict the market price to rise and others that predict it to fall.

5 Contextual Rationality of IKE Models

We have sketched how IKE’s approach to modeling market outcomes on the basis of mathematical microfoundations enables an economist to escape the insurmountable flaws inherent in extant approaches, including the gross irrationality of both REH and fully predetermined non-REH representations. However, the implications of jettisoning sharp predictions are broader. Acknowledging the limits to knowledge of how individuals think about the future calls for a reexamination of the very notions of rationality and purposeful decision-making. This rather large undertaking requires a separate treatment, but our foregoing discussion underscores a few key differences between IKE’s view of rationality and that of extant approaches.

Economists usually equate rationality—being able to justify one’s actions by an appeal to one’s objectives and reason²⁰—with self-interest. Consequently, they represent rational decision making on the part of an individ-

²⁰Paraphrased from Meriam-Webster’s Dictionary.

ual with the deployment of resources that maximizes her well-being. That much is common to all approaches, including IKE. Conventional economists, however, go much further. They believe not only that self-interest is a universal human trait, which they invoke as the main “reason” behind individual decision-making, but that they can adequately represent self-interested behavior with mechanical rules. Indeed, contemporary economists often use the same fully predetermined representations to explain individual self-interested behavior and aggregate outcomes over many decades or in different economies or markets.

Behavioral economists share both the conventional view of rationality and the belief that it can be represented with mechanical rules. This understanding leads them to diagnose the incompatibility between actual behavior and conventional representations of “rational” behavior as a symptom of market participants’ “irrationality.” They then proceed to use mechanical rules to fully prespecify the irrationality that they believe they have found.

Our discussion of IKE suggests that even if, on the margin, conventional models’ failures have something to do with market participants’ irrationality, attempting to build economic theory on exact representations of irrationality is as futile as attempting to build it on exact representations of rationality. The key premise of IKE is that no mechanical rule can adequately represent how purposeful individuals alter their decision-making and how they forecast the future consequences of their decisions. Thus, we should expect to find what behavioral economists have found: gross discrepancies between conventional representations of rational decision making and the way individuals actually behave.²¹

The key role of forecasting in rational decision making also implies that, even if an economist were able to attribute clear objectives to a market participant, he would still be unable to assess or represent exactly the participant’s rationality or irrationality.²² Rational decisions depend on forecasts of future market outcomes, which are not only a result of the actions of many individuals, but also depend on future economic policies, political developments, and institutional changes. Thus, even if individuals are presumed to be purely self-

²¹In chapter 13 of our book, we discuss a striking example of how the reliance on mechanical rules can lead to absurd conclusions. Economists have convinced themselves that it is possible to earn profits systematically by following a rule as simple as betting against the forward exchange rate; yet individuals in currency markets, who are handsomely rewarded for finding such profit opportunities somehow ignore the forward-rate rule.

²²Kay (2004) has called this fundamental difficulty “obliquity.” As he quipped, “no one will ever be buried with the epitaph ‘He maximized shareholder value,’ ...because even with hindsight there is no way of recognising whether the objective has been achieved.”

interested, how they deploy their resources depends at least as much on the social context as it does on their personal motivations. In a world of imperfect knowledge, rationality is always contextual.

5.1 Combining Insights from Economics and Other Disciplines

IKE is compatible with, though it does not require, the presumption that market participants are contextually rational. However, we cannot disentangle the complex and changing interdependence between an individual's motivations and her forecasts, which are shaped primarily by the context within which she makes decisions. As such, the characterization of contextual rationality with only a priori assumptions is out of reach of economic analysis.

In modeling individual behavior, therefore, economists must make use of empirical findings about how individuals actually behave. This necessity undermines the common belief among economists, which is increasingly echoed by others, that contemporary economics can rigorously explain the findings of other "soft" social sciences. The contextual view of rationality implies precisely the opposite: in order to represent purposeful individual decision-making, economists must draw on the findings of other social scientists. IKE makes use of these findings in specifying the microfoundations of its models.

5.1.1 Preferences

Many studies have found that conventional representations of preferences, which usually involve expected utility theory and the assumption of risk aversion, are grossly inconsistent with the way individuals actually behave. Much of the evidence concerning how individuals make choices is based on laboratory experiments in which the structure of payoffs from various gambles is predetermined by the experimenter. This common experimental design allows the investigator to examine the nature of an individual's preferences without the confounding problem of having to represent her forecasts of the potential payoffs from gambling. The seminal formulation of prospect theory by Kahneman and Tversky (1979) and Tversky and Kahneman (1992), including loss aversion, make use of such a experimental setup.

Although laboratory experiments have been the key to uncovering new ways to model preferences, their typical design effectively limits the economist's view of an individual's decision-making; the economist is able to observe only the subject's responses to an experimenter's stimuli. This basic framework, which is used extensively in psychological research, sidesteps a

key problem: participants in real-world markets forecast payoffs—the experimenter’s “stimuli”—on the basis of imperfect knowledge. Moreover, these forecasts depend not only on the subject’s creativity, her analytical abilities, and other personal characteristics, but also on the unfolding social context.²³ As a result, the basic type of model used in these psychological experiments is grossly insufficient as a foundation for representing economic behavior.

In specifying the microfoundations of our models of asset markets, we extend Kahneman and Tversky’s original formulation of prospect theory to recognize the importance of imperfect knowledge. Our formulation, which we call endogenous prospect theory, is consistent with the experimental evidence. This representation supposes that an individual’s utility ranking over alternative speculative positions in the market depends on her forecasts of the outcomes of these positions, in particular, on her forecasts of future returns and the potential losses that she might incur. Endogenous prospect theory also assumes that an individual’s degree of loss aversion increases as the her forecast of the size of potential losses increases. Because we represent forecasting with qualitative conditions, the way in which an individual’s degree of loss aversion changes between any two points in time is only partially predetermined in our models.

5.1.2 Forecasting Behavior

The premise that self-interested or, more broadly, purposeful behavior is to an important degree context-dependent does not preclude the usefulness of insights from psychology in modeling individual behavior. Indeed, we make use of some of these insights in representing how an individual revises her forecasting strategy.²⁴

However, the importance of the social context for an individual’s decision-making implies that, in searching for empirical regularities that might be useful in modeling an individual’s decisions, economists will need to look beyond laboratory experiments and insights from psychology. Other social scientists have knowledge and intuitions concerning the social context within which individuals make decisions that may complement economists’ work in modeling individual forecasting behavior.

We make use of Keynes’s (1936) insight that conventions among market

²³Kahneman and Tversky (1979) recognized that laboratory experiments, while useful in uncovering the properties of the utility function over single outcomes, may be much less informative about an individual’s choices over gambles with two or more uncertain outcomes in real-world markets.

²⁴For an example in the context of modeling currency swings, see section 12.1.

participants play an important role in individual decision-making. We also draw on our understanding of the qualitative regularities that have characterized aggregate outcomes; we suppose that market participants must also be aware of these regularities when they form their forecasts. For example, the tendency of exchange rates to undergo long swings away from historical benchmark levels and then to exhibit sustained counter-movements plays a key role in our model of the premium on foreign exchange.

The distinguishing feature of IKE models is that they require an economist to prespecify neither the potential set of causal variables that underpin change in outcomes nor the influences of these variables in his representation. This is important, as the presumption that an economist can prespecify, even partially, the set of causal variables and their influences is very bold.

Nevertheless, in addressing some problems, an economist may have to represent these aspects of the causal mechanism. For example, in order to examine whether macroeconomic fundamentals matter for exchange rate movements, an economist must prespecify, at least partially, a representation of the causal mechanism, which includes the set of potential fundamentals (potential causal factors) and how they influence the exchange rate.

To this end, we consider the possibility that the stock of extant economic models summarizes economists' insights concerning the causal factors that underpin market outcomes. Presumably, these insights are shared by market participants. This idea underlies the Theories Consistent Expectations Hypothesis (TCEH) proposed by Frydman and Phelps (1990). TCEH recognizes that a set of extant economic models at best indicates to a market participant, or to an economist attempting to represent her behavior, which causal variables may be important for forecasting market outcomes; it also suggests, in a qualitative way, how these variables may influence those outcomes.²⁵

In our book, we propose a simple procedure that enables an economist to take into account the qualitative features of more than one model in constructing his representation of market participants' forecasting strategies. We show that TCEH representations can rationalize many of the sign predictions implied by the REH monetary models of the exchange rate. However, although TCEH may seem to be just a qualitative analog of REH, there are two fundamental differences. In order to account for the social context within which market participants' act, TCEH recognizes that an economist cannot ignore

²⁵In an early implementation of TCEH, Goldberg and Frydman (1996a) relied on the qualitative features of the reduced forms of several REH models to represent individual forecasting behavior. In our book, we eschew REH and propose how an economist can decipher the qualitative features of the reduced forms of a set of models under imperfect knowledge.

the pluralism of models. Moreover, TCEH only partially prespecifies change.

6 IKE as the Boundary of What Macroeconomic Theory Can Deliver

In our book, we show how IKE models shed new light on the salient features of the empirical record on exchange rates, which have confounded international macroeconomists for decades. In part III, we sketch some of our analysis and show it can be applied to study price movements in other asset markets. Although these results are promising, it is much too early to claim broader usefulness for IKE in macroeconomic and policy modeling.

In contrast to the conventional approach, which seeks to understand economic decisions with universal mechanistic rules, the constraints of IKE models are qualitative and context dependent. If qualitative regularities can be established in contexts other than asset markets, IKE can show how they can be incorporated into mathematical models. However, in contexts in which revisions of forecasting strategies cannot be adequately characterized with reasonably long-lasting qualitative conditions, empirically relevant models of the observed time-series may be beyond the reach of economic analysis. In this sense, IKE provides the boundary to what modern macroeconomic theory — which aims to explain empirical regularities in aggregate outcomes with models that are based on mathematical microfoundations — can deliver.

Part II

Why Macroeconomic Theory Cannot Ignore the Limits to Knowledge

7 Fully Prespecifying Change to Generate Sharp Predictions

In this part, we make use of a simple algebraic model of the market price to formalize the key arguments of our critique of contemporary models. We begin by showing how the pursuit of sharp predictions leads economists to fully prespecify change in their models.

Our simple algebraic example is motivated by basic supply and demand analysis. In financial markets as well as in many other markets, the quantity demanded and supplied on the individual level is often thought to depend on the forecast of the future market price and a set of causal variables, such as

the money supply or real income. Aggregating over individuals and equating aggregate demand and supply, typically yields the following representation, in semi-reduced form, for the equilibrium market price at a point in time:

$$P_t = a_t + b_t X_t + c_t \hat{P}_{t|t+1} \quad (1)$$

where X_t is a set of causal variables, (a_t, b_t, c_t) is a vector of parameters, and $\hat{P}_{t|t+1}$ is an aggregate of market participants' forecasts formed at t of the market price at $t + 1$.

Individual forecasts that comprise the aggregate forecast, $\hat{P}_{t|t+1}$, are formed on the basis of forecasting strategies at t . Contemporary economists represent these forecasts by relating them to a set of causal variables, which represents the information sets used by market participants. An aggregate of such representations can be written as,

$$\hat{P}_{t|t+1} = \alpha_t + \beta_t Z_t \quad (2)$$

where Z_t is a vector of causal variables that characterizes the union of information sets used by market participants and (α_t, β_t) is a vector of parameters.

An economist formalizes his assumptions about individual decision-making and how it translates into aggregate outcomes by placing restrictions on his representations. At each point in time, the structure of an economist's model is characterized by the following properties:

1. The composition of the set of causal variables appearing in the representations on the individual and aggregate levels.
2. The properties of the joint probability distribution of the causal variables.²⁶
3. A functional form that relates outcomes to the causal variables, which typically includes the signs of partial derivatives. In cases such as our example, in which the functional form is explicit, economists often restrict the signs of some parameters.

In general, as time passes, individuals alter the way they make decisions. Various aspects of the social context also change. These changes influence the way aggregate outcomes move over time. Thus, to model the causal mechanism over time, an economist will need different structures—different specifications

²⁶If the the model includes additive error terms, the conditions imposed by an economist also specify the joint probability distribution between these terms and the causal variables.

of forecasting, preferences, constraints, decision and aggregation rules, or the processes driving the causal variables—at different points in time to represent individual behavior and market outcomes.

However, as we have already mentioned, most economists construct models that use the same structure to represent individual behavior and aggregate outcomes at every point in time. These time-invariant models presume that individuals never alter, let alone devise, new ways to forecast future market outcomes. Sometimes economists do recognize the need to allow for change in their models. However, the insistence that their models should generate sharp predictions leads both conventional and behavioral economists to impose restrictions that fully prespecify this change; that is, they relate the properties of the structure of a model at all points in time, past and future, exactly to the properties of its structure at some arbitrary “initial” point in time.

7.1 Fully Predetermining Restrictions

What is perhaps most striking about contemporary representations is that they prespecify exactly how individuals revise the way they think about the future and how the social context unfolds over time.

In general, to generate sharp predictions from his model, an economist would need to fully prespecify the timing of all revisions of forecasting strategies and their post-change representations. Such change could involve different sets of causal variables or even different functional forms. Because these complications would not affect any of our conclusions, we suppose that an economist represents revisions of forecasting strategies with a parametric shift in his aggregate representation at $t + 1$ and that he assumes that these strategies will remain unchanged thereafter:

$$\hat{P}_{t+\tau|t+\tau+1} = \alpha_{t+\tau} + \beta_{t+\tau} Z_{t+\tau} \quad (3)$$

where $\alpha_t \neq \alpha_{t+1}$ and $\beta_t \neq \beta_{t+1}$ and $\alpha_{t+\tau} = \alpha_{t+\tau+1}$, and $\beta_{t+\tau} = \beta_{t+\tau+1}$ for all $\tau = 1, 2, 3, \dots$. In this example, revisions, which are set to occur only at $t + 1$, are represented by two constants $A_{(t,t+1)}$ and $B_{(t,t+1)}$:²⁷

$$A_{(t,t+1)} = \alpha_{t+1} - \alpha_t \quad \text{and} \quad B_{(t,t+1)} = \beta_{t+1} - \beta_t \quad (4)$$

Contemporary economists fully prespecify revisions of forecasting strategies, which in the context of our example, would constrain $A_{(t,t+1)}$ and $B_{(t,t+1)}$

²⁷Except for purely formal complications, our conclusions in this section apply to nonlinear representations. For example, suppose that the representation of the aggregate forecasting strategy at $t + 1$ is a nonlinear function of the causal variables. In such a case, $A_{(t,t+1)}$ and $B_{(t,t+1)}$ would be nonlinear functions of the causal variables.

to take on particular values. The assumption of time-invariance would constrain the representation of forecasting strategies to remain unchanged at all times, past and future; that is it would set $A_{(t,t+1)} = 0$ and $B_{(t,t+1)} = 0$.

Infrequently, economists allow for revisions of forecasting strategies in their models. A simple example of how change can be fully predetermined is to set $A_{(t,t+1)}$ and $B_{(t,t+1)}$ equal to particular values \bar{A} and \bar{B} , respectively:

$$A_{(t,t+1)} = \alpha_{t+1} - \alpha_t = \bar{A} \quad \text{and} \quad B_{(t,t+1)} = \beta_{t+1} - \beta_t = \bar{B} \quad (5)$$

We refer to such restrictions, which relate a model's pre-and post-change structures, as *fully predetermining*. Sometimes fully predetermining restrictions are probabilistic. For example, an influential class of contemporary models makes use of a rule that fully prespecifies the timing of all changes and the functions that relate $A_{(t,t+1)}$ and $B_{(t,t+1)}$ to the values of the causal variables, conditional on the "initial" structure.²⁸

To complete the task of fully predetermining his model, an economist would follow the foregoing procedure and fully prespecify its other components. To simplify our example further and focus on change arising from revisions in forecasting strategies, we follow much of the literature and assume that these other components are time-invariant. This assumption implies the following fully predetermining restrictions in (1):

- The composition of the set causal variables, X_t , and the properties of their joint probability distribution remain unchanged at all times, past and future.
- The parameters (a_t, b_t, c_t) are constants, that is, $(a_t, b_t, c_t) = (a, b, c)$ for all t .

By imposing the foregoing fully predetermining restrictions on (1) and (2), an economist presumes that the following model adequately characterizes the aggregate of forecasting strategies and the market price at $t + 1$ and beyond:

$$\hat{P}_{t+\tau|t+\tau+1} = (\alpha_t + \bar{A}) + (\beta_t + \bar{B})Z_{t+\tau} \quad \text{for } \tau = 1, 2, 3... \quad (6)$$

$$P_{t+\tau} = (a + c(\alpha_t + \bar{A})) + bX_{t+\tau} + c(\beta_t + \bar{B})Z_{t+\tau} \quad \text{for } \tau = 1, 2, 3... \quad (7)$$

²⁸For example, see Hamilton (1989, 1994). Frydman and Goldberg (2007, chapter 6) show that all of our conclusions in this section apply to models that use a probabilistic rule to fully prespecify the timing of revisions and the post-change forecasting strategies.

7.2 Sharp Probabilistic Predictions of Change

The model of the causal mechanism driving outcomes in (6) and (7) enables an economist to generate sharp probabilistic predictions of future outcomes. In order to do so, an economist must fully prespecify the movement of the causal variables over time. To keep our example simple, we assume that all causal variables follow a random walk with constant drift,

$$X_t = \mu^X + X_{t-1} + \varepsilon_t^X \quad (8)$$

$$Z_t = \mu^Z + Z_{t-1} + \varepsilon_t^Z \quad (9)$$

where $\varepsilon_t = (\varepsilon_t^X, \varepsilon_t^Z)$ is an i.i.d. vector of random errors and $E[\varepsilon_t^X] = E[\varepsilon_t^Z] = 0$.²⁹

To illustrate the concept of sharp probabilistic predictions of change on the individual level, we shift (6) and (8) one period ahead. Using (2) implies the following representation of revisions of forecasting strategies:

$$\Delta \hat{P}(t+1, t) = \hat{P}_{t+1|t+2} - \hat{P}_{t|t+1} = [\bar{A} + (\beta_t + \bar{B})\mu^Z] + (\beta_t + \bar{B})\varepsilon_t^Z \quad (10)$$

The model also implies the following representation of change in market outcomes between t and $t+1$:

$$\Delta P(t+1, t) = P_{t+1} - P_t = [c\bar{A} + b\mu^X + c(\beta_t + \bar{B})\mu^Z] + [b\varepsilon_t^X + c(\beta_t + \bar{B})\varepsilon_t^Z] \quad (11)$$

The expressions in (10) and (11) show that fully predetermined models represent forecasting strategies and aggregate outcomes as unfolding over time around deterministic time paths— $[\bar{A} + (\beta_t + \bar{B})\mu^Z]$ and $[c\bar{A} + b\mu^X + c(\beta_t + \bar{B})\mu^Z]$ —conditional on the structure of the model at some arbitrary point in time— α_t , β_t , a , b , c , μ^X , and μ^Z . Although contemporary models allow for deviations from these paths— $(\beta_t + \bar{B})\varepsilon_t^Z$ and $[b\varepsilon_t^X + c(\beta_t + \bar{B})\varepsilon_t^Z]$ —they fully prespecify how the probability distributions of these deviations might change over time. As is usual in the literature, our example assumes that these distributions are invariant.³⁰ Thus, as different as they may appear, fully predetermined probabilistic representations of change are as restrictive as their deterministic counterparts, linear or nonlinear: they both make no allowance whatsoever for the possibility that at some point, individuals decide to think about the future in ways that could not have been foreseen in advance or that policy officials alter policy in new ways.

²⁹Such random walk specifications are popular in macroeconomic models that involve variables such as the money supply, income, interest rates, and the price level.

³⁰See Engle (2003) for a specification that allows for fully predetermined changes in the distributions of error terms.

7.2.1 History as the Future and Vice Versa

Another way to see how odd this conception of change is, note that contemporary models represent history as if it were fully reversible. To illustrate this point, consider the representation of forecast revisions in (10) and suppose that one of the causal variables increases as time passes from t to $t + 1$. An individual is supposed to revise her forecast over this period in a way that is fully prespecified by an economist. If the causal variable then subsequently declines back to its original value between $t + 1$ and $t + 2$, market participants are presumed to revert to the same forecast as the one they started with at t .

Although this conclusion follows immediately from the invariance restriction, it also holds in a model in which the change in the causal variable triggers a fully prespecified revision of forecasting strategies. However, as soon as the causal variable returns to its initial value, forecasting strategies also revert to their initial representations.^{31,32} This representation of change is tantamount to presupposing that as time passes, market participants do not gain any experience or come up with any genuinely new ways of thinking about the future. Remarkably, the contemporary approach presumes that the passage of time does not play an essential role in altering individual behavior and the social context within which individuals make decisions.

8 Epistemological Flaws of Contemporary Models in a World of Imperfect Knowledge

8.1 Internal Inconsistency When Forecasting Strategies Are Diverse

Hayek (1945) posited that one cannot understand how markets allocate resources without considering the central role played by the division of knowledge, which is a key factor behind diversity of forecasting strategies. Yet, a

³¹See Frydman and Goldberg (2007, chapter 5) for a rigorous demonstration in the context of demand and supply analysis.

³²We note that change in multiple-equilibrium models is not, in general, reversible: a return of a causal variable to its initial value could be associated with a move to an equilibrium other than the initial one. Economists sometimes construct so-called hysteresis models in which the movement of an outcome variable is path-dependent. For example, see Krugman (1987). Nevertheless, both types of models fully prespecify the equilibria to which the system could move. Consequently, they both presume that individuals have not invented any genuinely new ways of thinking about the future as history unfolds. See Frydman and Goldberg (2007, chapter 6) and references therein.

fully predetermined model that allows for such diversity is necessarily internally inconsistent.

To illustrate this point, suppose that the model in (1) adequately represents the causal mechanism that underpins the behavior of the equilibrium price. Also suppose that in modeling the microfoundations of (1), an economist recognizes that market participants make use of diverse forecasting strategies, which for simplicity we represent with two strategies, $\hat{P}_{t|t+1}^{(1)}$, and $\hat{P}_{t|t+1}^{(2)}$. To generate sharp predictions, the representations of these two strategies must be fully predetermined. Following the literature, and without loss of generality, we constrain these representations to be time-invariant:

$$\hat{P}_{t|t+1}^{(1)} = \alpha^{(1)} + \beta^{(1)} Z_t^{(1)} \quad (12)$$

and

$$\hat{P}_{t|t+1}^{(2)} = \alpha^{(2)} + \beta^{(2)} Z_t^{(2)} \quad (13)$$

where, $Z_t^{(i)}$ $i = 1, 2$ are vectors of causal variables that represent the information sets used by market participants in each group and $(\alpha^{(i)}, \beta^{(i)})$, $i = 1, 2$ are vectors of parameters. Diversity of forecasting strategies implies that $\alpha^{(1)} \neq \alpha^{(2)}$ and $\beta^{(1)} \neq \beta^{(2)}$. The aggregate forecast appearing in (1) can then be represented as an average of market participants' forecasts:

$$\hat{P}_{t|t+1} = \omega \hat{P}_{t|t+1}^{(1)} + (1 - \omega) \hat{P}_{t|t+1}^{(2)} \quad (14)$$

where the weight $0 < \omega < 1$ represents the importance of type 1 individuals in influencing the behavior of the equilibrium price.

Substituting (14) into (1) yields the following representation of the causal mechanism that drives the market price:

$$P_t = a + bX_t + c [\omega \alpha^{(1)} + (1 - \omega) \alpha^{(2)}] + c [\omega \beta^{(1)} Z_t^{(1)} + (1 - \omega) \beta^{(2)} Z_t^{(2)}] \quad (15)$$

It is apparent that the model is internally inconsistent in the sense articulated by Lucas: the representations of individual forecasting strategies in (12) and (13) are systematically inconsistent with the representation of the market price in (15). To see this, assume that the causal variables follow random walks with constant drifts, analogous to (8) and (9). This implies that the difference between the aggregate representation and the representation for, say, type 1 individuals can be written as:

$$P_{t+1} - \hat{P}_{t|t+1}^{(1)} = C + bX_t + (c\omega - 1) \beta^{(1)} Z_t^{(1)} + c(1 - \omega) \beta^{(2)} Z_t^{(2)} + \eta_t \quad (16)$$

where C is a non-zero constant that depends on drift terms and η_t is an i.i.d. random variable that depends on error terms from the X , Z^1 , and Z^2 processes. As we discussed in section 3.3, such representations imply that individuals systematically forego obvious profit opportunities.

8.2 REH as a Representation of Grossly Irrational Forecasting Strategy

REH models avoid internal inconsistency by disregarding the importance of diversity of forecasting strategies. Of course, self-interested, rational individuals would collectively adhere to one forecasting strategy in perpetuity only if “all agents have solved their ‘scientific problems’” (Sargent 1993, p.23). In such an imaginary world, REH would be a plausible hypothesis. All market participants and economists would have discovered an overarching causal mechanism that characterizes aggregate outcomes, as well as how the causal factors unfold over time, and thus, individual creativity and, more broadly change that does not follow pre-existing rules, would cease to be economically important. Economic decisions would become purely routine and passive, and thus capable of being captured by fully predetermined representations. In this fanciful world, contemporary representations would adequately explain individual behavior and would be completely consistent with the model of aggregate outcomes. Moreover, in such an *REH world*, the heterogeneity of forecasts among market participants would stem solely from differences in information.

Of course, in the real world, where the scientific problem has not been solved, there is a division of knowledge among individuals. Market participants forecast not only on the basis of different factors (their information sets), but also on the basis of different strategies (their knowledge) that map these factors into forecasts. No one knows—because no one *can know*—precisely how knowledge differs among individuals.

By design, REH models are fully predetermined. Thus, if REH models were to recognize the diversity of forecasting strategies, they would be internally inconsistent. In positing that inconsistent models are the wrong theory, Lucas (2001, p.13) argued that they represent individual market participants as *grossly irrational*, in so far as they disregard *endlessly* systematic information in their forecast errors. It is easy to show that this conclusion applies to REH models in a world of imperfect knowledge.

To this end, we write the REH representation of the representative individual’s forecasting strategy as a linear function in X_t :

$$\widehat{P}_{t|t+1}^{\text{RE}} = \alpha^{\text{RE}} + \beta^{\text{RE}} X_t \quad (17)$$

where the superscript “RE” denotes an REH representation. REH instructs an economist to determine his individual and aggregate representations *jointly*. Consequently, while an REH theorist would specify individual preferences and constraints autonomously from his aggregate model, his representation of an individual’s forecasting behavior is derivative of these other components. The causal variables X_t and the parameters $(\alpha^{\text{RE}}, \beta^{\text{RE}})$ of the forecasting strategy in (17) stem from the representation of preferences, the social context, and the constraints that an individual faces in making her decisions. In this way, REH rules out an autonomous role for market participants’ knowledge in shaping market outcomes.

To derive the REH representations, an economist chooses the coefficients α^{RE} and β^{RE} to be functions of the parameters a , b , and c in (1) to ensure the required consistency between the representations on the individual and aggregate levels:

$$\widehat{P}_{t|t+1}^{\text{RE}} = E[P_{t+1}^{\text{EM}} | X_t] \text{ for all realizations of } X_t \quad (18)$$

where the superscript “EM” denotes the representation implied by an economist’s model and $E[[]]$ is the expectation of P_{t+1}^{EM} , conditional on X_t and the constraint that the structure in (1) is time-invariant.

Imposing (18) in (1), and using (8), the model implies the following REH representation for the market price at $t + 1$ and the average—representative—forecast of this price formed at t :³³

$$P_{t+1}^{\text{RE}} = \frac{a(1-c) + b\mu^X}{(1-c)^2} + \frac{b}{(1-c)} X_t + \epsilon_{t+1} \quad (19)$$

$$\widehat{P}_{t|t+1}^{\text{RE}} = \frac{a(1-c) + b\mu^X}{(1-c)^2} + \frac{b}{1-c} X_t \quad (20)$$

where $\epsilon_{t+1} = \frac{b}{1-c} \varepsilon_{t+1}^X$.

Now suppose, as in fact every REH theorist does, that (19) and (20) represent adequately the causal mechanism that underpins the time-path of the market price and the representative forecast, respectively. Moreover, we follow the usual interpretation of a representative individual’s forecasting strategy

³³For details of the derivation of the REH solution, see chapter 3 in Frydman and Goldberg (2007).

and assume that the representation in (20) stands for an aggregate of representations of individual behavior. Without loss of generality, we write this aggregate forecast analogously to (14),

$$\widehat{P}_{t|t+1}^{\text{RE}} = \omega_1 \widehat{P}_{t|t+1}^{(1)} + \omega_2 \widehat{P}_{t|t+1}^{(2)} \quad (21)$$

The REH representation in (21) is assumed to represent the average of forecasts across market participants. Thus, the fully predetermined representations of the individual forecasting strategies are

$$\widehat{P}_{t|t+1}^{(1)} = \alpha^{(1)} + \beta^{(1)} X_t \quad (22)$$

$$\widehat{P}_{t|t+1}^{(2)} = \alpha^{(2)} + \beta^{(2)} X_t \quad (23)$$

where again the diversity of forecasting strategies implies $\alpha^{(1)} \neq \alpha^{(2)}$ and $\beta^{(1)} \neq \beta^{(2)}$.³⁴

This REH model also implies the following representation of the forecast errors for type 1 individuals:

$$f e_{t|t+1}^{1,\text{RE}} = P_{t+1}^{\text{RE}} - \widehat{P}_{t|t+1}^{1,\text{RE}} = C' + \left[(b - \beta^{(1)}) + c\omega(\beta^{(1)} - \beta^{(2)}) + c\beta^{(2)} \right] X_t + \epsilon'_{t+1} \quad (24)$$

where again C' depends on drift terms and ϵ'_{t+1} depends on the error terms for the X_{t+1} process. Thus, in a world in which individuals have not solved their scientific problems, but where an REH model nonetheless adequately represents the behavior of market outcomes, the REH representation of an individual's forecasting strategy implies that she disregards endlessly the obvious systematic information contained in her forecast errors. To paraphrase Lucas (2001, p.13), the REH model would imply that there are profit opportunities that a market participant could see. If she does see these opportunities and she is rational, she would revise her forecasting strategies. Not doing so—ever—would be grossly irrational.

8.3 Behavioral Models

Despite its epistemological implausibility and empirical failures, some behavioral economists continue to rely on REH to represent individual forecasting. In contrast to conventional methodology, however, the behavioral approach

³⁴These parameters must satisfy the restrictions implied by the representations in (20) through (23).

does not oblige an economist to use REH. This has led some behavioral economists to develop non-REH representations of market participants forecasting strategies, which are autonomous from their aggregate models. Departing from REH has also enabled behavioral economists to recognize the role of diversity in driving market outcomes.³⁵

8.3.1 Internal Inconsistency

However, behavioral economists embrace the conventional insistence on sharp predictions and thus fully predetermine their non-REH models. Even if they do not allow for diversity, fully predetermined models that jettison REH are internally inconsistent.

To illustrate this point, suppose that on the basis of some empirical considerations, a behavioral economist specifies a time-invariant version of the non-REH representation in (2):³⁶

$$\hat{P}_{t|t+1} = \alpha + \beta Z_t \quad (25)$$

Substituting (25) into the time-invariant version of (1) and shifting it one period ahead yields:

$$P_{t+1} = [a + c\alpha + b\mu^X + c\beta\mu^Z] + bX_t + c\beta Z_t + [b\varepsilon_t^X + c\beta\varepsilon_t^Z] \quad (26)$$

where we used the representations for X_{t+1} and Z_{t+1} in (8) and (9), respectively.

Comparing the representation on the individual level in (25) with that on the aggregate level in (26) immediately implies that for $(\alpha, \beta) \neq (\alpha^{\text{RE}}, \beta^{\text{RE}})$, the fully predetermined behavioral model is internally inconsistent. As Lucas argued, such internally inconsistent models are “the wrong theory” of time-series regularities.

The fully predetermined microfoundations of the model presume that on average market participants are grossly irrational. This is easily seen by subtracting (25) from (26):

$$fe_{t|t+1} = P_{t+1} - \hat{P}_{t|t+1} = D + bX_t + (c - 1)\beta Z_t + \varepsilon_{t+1} \quad (27)$$

³⁵One of the earliest examples that allowed for diversity of forecasting strategies, and that anticipated the behavioral approach, is the seminal exchange-rate model of Frankel and Froot (1987).

³⁶Our argument remains valid for behavioral models that allow for change, but fully respecify it. See chapter 6 of our book.

where again D depends on drift terms and ϵ_{t+1} depends on the error term for the X_{t+1} and Z_{t+1} processes. The expression in (27) shows that the model attributes to a representative individual a forecasting strategy that generates systematic forecast errors *endlessly*. Thus, although non-REH behavioral models recognize that market participants' forecasts are "inputs" to rather than "outputs" of economic models, their microfoundations are implausible. Moreover, they retain the key feature of REH models: they presume that market participants never devise ways to forecast the future that an economist cannot specify in advance.

9 Why Macroeconomics Must Open Its Models to Imperfect Knowledge

We now show that to escape the insurmountable epistemological difficulties inherent in extant approaches, it is *necessary* for economic models to stop short of fully prespecifying change. This is tantamount to acknowledging that sharp predictions are outside the reach of economic analysis and that the most economic theory can be expected to deliver is qualitative predictions of market outcomes.

Consider again the model in equations (12), (13), and (15). For simplicity, we continue to impose the invariance restriction in (1), but drop this assumption for the representations of market participants' forecasting in (12) and (13). We write the representations of these strategies at time $t + 1$ in terms of their structure and the realizations of the causal variables appearing in them at time t :

$$\hat{P}_{t+1|t+2}^{(1)} = [\alpha_t^{(1)} + A^{(1)}(t, t + 1)] + [\beta_t^{(1)} + B^{(1)}(t, t + 1)]Z_{t+1}^{(1)} \quad (28)$$

and

$$\hat{P}_{t+1|t+2}^{(2)} = [\alpha_t^{(2)} + A^{(2)}(t, t + 1)] + [\beta_t^{(2)} + B^{(2)}(t, t + 1)]Z_{t+1}^{(2)} \quad (29)$$

where $A^{(i)}(t, t + 1) = \alpha_{t+1}^{(i)} - \alpha_t^{(i)}$ and $B^{(i)}(t, t + 1) = \beta_{t+1}^{(i)} - \beta_t^{(i)}$, $i = 1, 2$. Analogously to (15), the model implies the following representation of the causal mechanism for the market price at time $t + 1$:

$$\begin{aligned}
P_{t+1} = & a + bX_t \\
& + \left\{ c\omega[\alpha_t^{(1)} + A^{(1)}(t, t+1)] + (1-\omega)[\alpha_t^{(2)} + A^{(2)}(t, t+1)] \right\} \\
& + c \left\{ \omega[\beta_t^{(1)} + B^{(1)}(t, t+1)]Z_t^{(1)} + (1-\omega)[\beta_t^{(2)} + B^{(2)}(t, t+1)]Z_t^{(2)} \right\} \\
& + \eta_t
\end{aligned}$$

where η_t depends on the error terms for the processes representing the causal factors X_t and Z_t . It is clear that to avoid inconsistency between the predictions of any one of the diverse forecasting strategies allowed for by (28) and (29) and those of the representation of the market price, the aggregate representation in (??) must be compatible with more than one conditional probability distribution.³⁷

10 IKE: Recognizing Imperfect Knowledge and Diversity of Forecasting Strategies

10.1 Forecasting As Entrepreneurship

The contemporary approach presumes that economists can represent exactly how individuals' decision-making, particularly their forecasting strategies, and aggregate outcomes unfold over time. If true, then understanding the past would be a relatively straightforward problem: all one would need to do is to specify an appropriate econometric model and estimate it on the basis of as much past data as was available. As the future in a contemporary model follows exactly from the past, except for "sampling error," the estimated econometric model could be used to forecast market outcomes.

Purposeful individuals, however, recognize that even when it comes to the past, everyone has only imperfect knowledge about the timing of changes in the causal mechanism driving outcomes. It is unclear, therefore, how to implement the econometric strategy suggested by the contemporary approach: the use of past data to estimate a model requires an assessment of how far back in time it adequately represents the economy. Even the most sophisticated statistical techniques would not automatically pinpoint when the last structural break

³⁷All probability distributions of future outcomes that we refer to are conditional on the structure of the model and the realizations of the causal variables at some prior point in time, which in the present case is t .

had occurred.³⁸ Thus, to understand the past causal mechanism one cannot merely estimate some model using all the available data and presume that it adequately represents historical outcomes. Unsurprisingly, interpretations of the past vary among individuals even when they use formal methods.

Moreover, even if we could adequately interpret the past with a fully pre-determined model, the future would still remain imperfectly known. Because future outcomes depend on revisions of forecasting strategies that cannot be predicted “by rational and scientific methods” (Popper, 1957, p. xii), market participants do not merely rely on pre-existing rules, whether based on formal or informal considerations, to forecast future changes in the causal mechanism. Ultimately, good forecasting is much like successful entrepreneurship: it may involve the use of quantitative models, but it also relies on one’s own personal knowledge, intuition, and a bit of luck in spotting profit opportunities.

For example, consider the problem of forecasting exchange rates. Many market participants form exact forecasts of the future exchange rate, for example, that a euro will cost \$1.5 in a week. After all, a currency trader must decide on her market position at each point in time. Nevertheless, although a market participant may base her trading on exact predictions, she does not arrive at such predictions by relying solely on quantitative models, much less the same model in every time period. In forming her forecasts, a purposeful individual often combines *her preferred* quantitative model with her own insights concerning the behavior of other market participants, the historical record on exchange rate fluctuations, and her evaluation of the impact of past and future decisions by policy officials.³⁹ Moreover, because market participants act on the basis of different experiences, interpretations of the past, and intuitions about the future, they adopt diverse strategies in forming and revising their exchange rate forecasts over time.

10.2 Imperfect Knowledge and Economic Analysis

Even though individuals in real world markets combine both formal and informal methods in forming their forecasts, IKE recognizes that representing their behavior in a mathematical model requires an economist to find some way to

³⁸The results of statistical tests of a model’s invariance depend on the significance level of these tests, which involve an element of judgement on the part of the forecaster or an economist. See chapter 15 of our book.

³⁹In a world of imperfect knowledge, even the use of quantitative models alone involves subjectivity, because there is more than one way to represent the causal mechanism mathematically. One important reason is that, as in (1), economic models typically involve representations of market participants’ forecasting strategies. See chapter 15 of our book.

formalize it. Like any scientific theory, IKE strives to generate implications that have empirical content. As a result, it must presume that purposeful decision-making, although driven by a variety of factors that cannot be made fully intelligible by the individuals themselves, let alone by economists, nevertheless exhibits some regularities. Otherwise, no macroeconomic theory that can be confronted with time-series evidence would be possible. IKE searches for these regularities and formalizes them with the aid of probabilistic formalism.

10.2.1 Representing Imperfect Knowledge with Partially Predetermined Probability Distributions

In the context of our simple example, the constants $A^{(i)}(t, t+1)$ and $B^{(i)}(t, t+1)$, $i = 1, 2$ in (28), (29) and (??) represent revisions of forecasting strategies and the resulting change in the causal mechanism on the aggregate level. A contemporary economist would fully prespecify change by imposing quantitative conditions on these constants, typically setting $A^{(i)}(t, t+1) = 0$ and $B^{(i)}(t, t+1) = 0$. Such fully predetermining restrictions, together with stochastic representations of the causal factors, as in (8) and (9), for example, imply that the model represents the market price at $t+1$ with a single probability distribution, conditional on the structure and the values of the causal variables at t . It also represents the consequences of individual decisions with a single probability distribution.

By contrast, as we discussed in section 4.1.1, Knight and Keynes forcefully argued that business decisions are fraught with radical uncertainty, a situation in which neither market participants nor economists are able to represent any aspects of the causal mechanism that underpins change.

In the context of (28)-(??), such extreme interpretations of radical uncertainty can be formalized by not imposing any constraints on $A^{(i)}(t, t+1)$ and $B^{(i)}(t, t+1)$, $i = 1, 2$. Without constraints, equations (28)-(??) represent change with myriad of conditional probability distributions. Within the confines of our simple linear example, such representations imply that the model in (??) is compatible with *any* relationship between the causal variables and the market price. In this extreme version, radical uncertainty inherently conflicts with economists' attempts to distinguish among alternative causal explanations of market outcomes.

Although it does not impose fully predetermining restrictions on change, IKE aims to explain outcomes with mathematical models that can be confronted with empirical evidence. To this end, IKE uses probabilistic formal-

ism to formulate its mathematical representations of forecasting strategies and their revisions.

IKE thus adopts an intermediate position between radical uncertainty, which denies the possibility that economists might be able to formulate testable mathematical models of any features of the causal mechanism driving change, and the contemporary presumption that this mechanism can be adequately represented with a standard conditional probability distribution. Because IKE recognizes that everyone has access to only imperfect knowledge, it represents individual behavior and aggregate outcomes with many conditional probability distributions at every point in time. To model change, IKE constrains the possible transitions across these distributions to exhibit common qualitative properties. As we show in part III, the qualitative predictions concerning how the moments of the partially predetermined probability distributions unfold over time constitute the empirical content of an IKE model, in other words, that which can be confronted with time-series data.

Part III

Imperfect Knowledge Economics of Market Fluctuations and Risk

Market outcomes often undergo protracted swings that revolve around historical benchmark levels. For example, persistent movements in the unemployment rate away from a longer-term trend, or in stock prices away from levels consistent with historical price-earnings (PE) ratios, occur for years at a time. But the instability in the economy is bounded; eventually economic outcomes undergo sustained countermovements back to benchmark levels, which themselves vary over time. If outcomes happen to reach these levels, they often shoot through them. Moreover, although fluctuations are a recurrent feature of market outcomes in capitalist economies, the observed swings are uneven: the magnitude and duration of upswings and downswings vary from one episode to the next in a way that does not seem to follow any consistent pattern.

Economists have grappled for decades with trying to understand the causal mechanism driving fluctuations in capitalist economies. Some contemporary models explain market fluctuations not as movements away from the benchmark, but as movements of the benchmark itself. In these models, benchmark levels unfold along fully prespecified time paths. Thus, to explain fluctuations, economists have allowed for shifts in these time paths, stemming from changes

in technology, policy, or knowledge. While these factors undoubtedly play a role in how benchmarks might vary over time, their influences cannot be fully prespecified.⁴⁰

The movement of the imperfectly known benchmark is likely to play a significant role in some markets. However, in his paper we focus on the fluctuations around benchmark levels.⁴¹

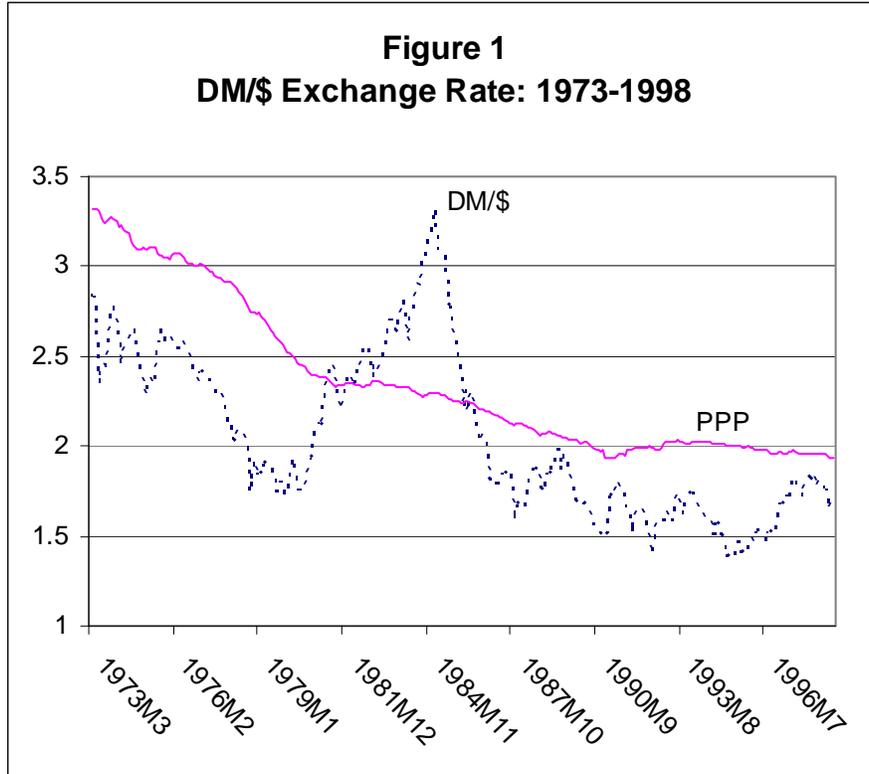
Figures 1 and 2 provide just two examples of the importance of viewing fluctuations in capitalist economies as swings around benchmark levels, rather than as movements of the benchmarks themselves. In figure 1, we plot the German mark-U.S. dollar (DM/\$) exchange rate and its purchasing power parity (PPP) level,⁴² while in figure 2, we plot the price of the Standard and Poor's 500 basket of stocks relative to a measure of its underlying earnings and a 20-year moving average of this PE ratio.⁴³ Both figures show prolonged periods in which the asset price tends to move persistently away from its benchmark. The figures also show that these price movements are uneven and ultimately bounded: the duration and magnitude of swings away from benchmark levels show no obvious pattern, but eventually they are followed by extended periods of time in which the asset price moves back toward these levels.

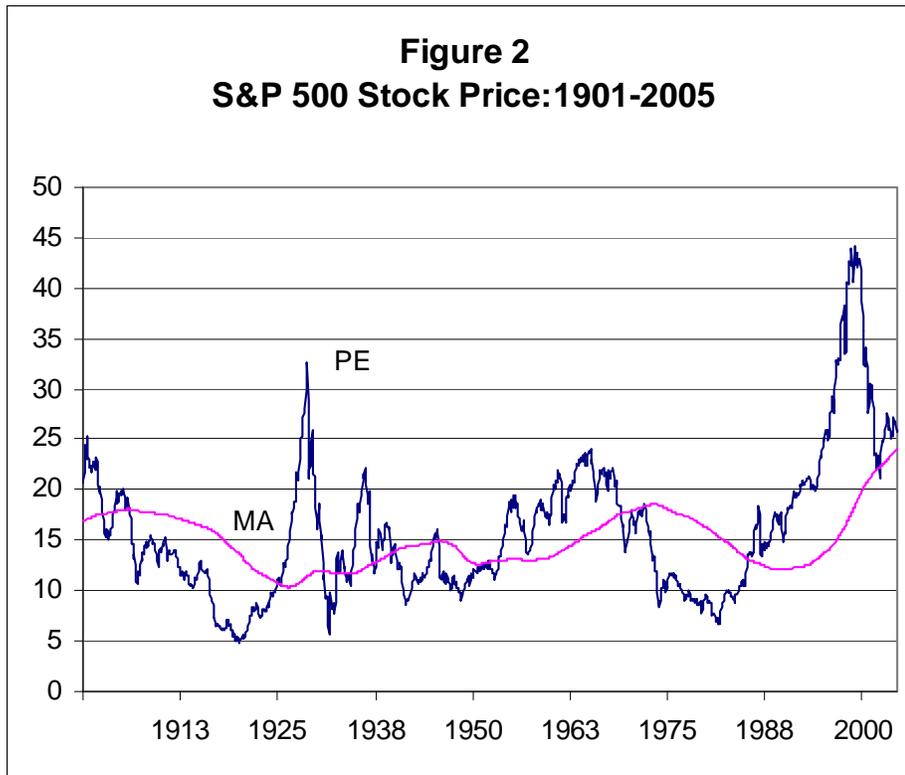
⁴⁰See Phelps (2008) for an argument that movements of the natural rate cannot be fully prespecified in capitalist economies.

⁴¹A model in which price movements arise from both fluctuations in the imperfectly known benchmark and swings away from the benchmark is beyond the scope of this paper.

⁴²The PPP benchmark in the figure is based on the *Big Mac* PPP exchange rate reported in the April 1990 issue of *The Economist* magazine (which was 1.96) and CPI-inflation-rate differentials from the IMF's *International Financial Statistics*.

⁴³Figure 2 is based on data from Shiller (2000), which are updated on his web site: www.econ.yale.edu/~shiller. PPP and average PE ratios have long traditions as benchmark levels in currency and stock markets, respectively.





In the remainder of this paper, we focus our attention on fluctuations in asset markets. In these markets, participants' forecasts play a key role in driving movements in prices away from and toward benchmark levels. Beyond modeling forecasting, we also sketch a new model of risk to help explain why long swings are ultimately bounded. Although we develop our IKE analysis in the context of asset markets, particularly those for currencies, our approach may prove useful in modeling fluctuations in other contexts in which forecasts are important in driving outcomes. However, as with the foreign exchange market, applying IKE in other markets will require that an economist search for context-specific representations of how forecasts in those markets might develop over time.

11 The Failure of REH Models

It is widely known that extant REH models are unable to account for the long-swings behavior of asset prices.⁴⁴ In currency markets, for example, flexible-

⁴⁴For early studies of the inconsistency of REH models with swings in stock and bond markets, see Shiller (1979, 1981, 1990). For an extensive discussion of the empirical failures

price REH models represent exchange rate dynamics as movements of the benchmark real-exchange rate. In these models, the benchmark depends on preferences and technology.⁴⁵ Thus, to account for the swings that we observe in figure 1, flexible price-models must assume that movements in these factors, whether stochastic or deterministic, undergo the kind of swings we observe in the exchange rate. But, exchange rate swings reverse direction much too often and abruptly for shifts in preferences and technology to provide a plausible explanation.⁴⁶

Another influential class of REH models, which assume that goods prices and/or wages adjust only sluggishly over time to equilibrium levels, view exchange rate dynamics as deviations from the PPP benchmark.⁴⁷ However, because REH ignores the role of autonomous revisions in forecasting strategies in driving outcomes, these sticky-price models are also unable to explain exchange rate fluctuations. By design, REH rigidly ties representations of individual forecasting behavior to the PPP: all market participants are assumed to predict a movement in the exchange rate back toward parity whenever a deviation from this benchmark arises. Random shocks in the model can push the exchange rate away from PPP if they happen to be of the same sign and of sufficiently large magnitude over successive points in time. However, long lasting runs of such random shocks are too improbable to explain the long swings away from PPP in figure 1.⁴⁸

Canonical REH models rule out an autonomous role for forecasts. Consequently, regardless of whether these models assume that prices are sticky or flexible, they must ignore the possibility that it is market participants' forecasts that push the exchange rate persistently away from PPP.

in currency markets, see Frydman and Goldberg (2007, chapter 7). REH models have also failed to explain excess returns in asset markets as risk premiums. In chapters 8 and 12 of our book, we discuss the inability of REH models to account for risk in currency markets.

⁴⁵See, for example, Stockman (1980) and Lucas (1982).

⁴⁶For criticism of flexible-price models of exchange rate swings along these lines, see Dornbusch (1989).

⁴⁷See, for example, Dornbusch (1976), Frankel (1979), and Obstfeld and Rogoff (1995).

⁴⁸Economists sometimes drop the assumption of stability and rely on the bubble paths of REH models to account for exchange rate swings. As we show in chapter 7 of our book, beyond sharing epistemological flaws with canonical REH models, these REH-bubble models are unable to account for the kind of long swings we observe in asset markets.

11.1 Imperfect Knowledge and Deviations from the REH Solution

Fully predetermined models, let alone their extreme REH versions, are inconsistent with the revisions and diversity of forecasting strategies that characterize real world markets. Moreover, economists themselves have constructed many different models. Thus, the aggregate of market participants' forecasting strategies differs from the strategy that is implied by any particular REH model, such as the one in equation (1). This divergence opens up the possibility that autonomous revisions of forecasting strategies play the key role in understanding long swings in asset markets.

To explore this possibility, consider again the semi-reduced form for the equilibrium price in equation (1), which we rewrite as⁴⁹

$$P_t = P_t^{\text{RE}} + c \left(\hat{P}_{t|t+1}^{\text{IK}} - \hat{P}_{t|t+1}^{\text{RE}} \right) \quad (30)$$

where, as before, $\hat{P}_{t|t+1}^{\text{RE}}$ is the REH forecast and $\hat{P}_{t|t+1}^{\text{IK}}$ denotes an IKE representation of the aggregate of individuals' forecasts.

Equation (30) shows immediately that P_t undergoes a protracted swing away from P_t^{RE} during periods of time in which $\hat{P}_{t|t+1}^{\text{IK}}$ moves persistently away from $\hat{P}_{t|t+1}^{\text{RE}}$. These periods end when the swing in $\hat{P}_{t|t+1}^{\text{IK}}$ ends.

11.2 Long Swings from Benchmark Levels in Asset Markets

This simple framework shows that allowing for imperfect knowledge has the potential to account for swings.⁵⁰ This, of course, presupposes that equation (1) provides an adequate semi-reduced-form representation of the equilibrium price.

This semi-reduced form has been used extensively to model asset prices, particularly those for stocks and currencies.⁵¹ Under the former interpretation, P_t is the price of a stock, X_t is the dividend paid each period on that stock, and $b = c$ is a discount factor. In the case of currencies, P_t is the logarithm of

⁴⁹Plugging $\hat{P}_{t|t+1}^{\text{RE}}$ into equation (1) gives $P_t = a + bX_t + c\hat{P}_{t|t+1}^{\text{RE}} + c \left(\hat{P}_{t|t+1} - \hat{P}_{t|t+1}^{\text{RE}} \right)$. Equation (30) follows from the fact that $P_t^{\text{RE}} = a + bX_t + c\hat{P}_{t|t+1}^{\text{RE}}$.

⁵⁰Our IKE model of long swings builds on some of the ideas of Schulmeister (1983, 1987) and Soros (1987).

⁵¹The extent to which the simple model in equation (1), together with an IKE representation of forecasting behavior, applies in markets beyond those for equity and currency is an open question that we do not explore in this paper.

the exchange rate, X_t consists of log levels of relative (domestic minus foreign) money supply and income, and b and c depend on the interest elasticity of money demand.⁵²

The specification in equation (30) recognizes the importance of imperfect knowledge. However, to explain swings around benchmark levels with this specification, the REH solution would have to correspond to an empirically-plausible notion of the benchmark in the particular market under study.⁵³

As it turns out, this seems to be the case in equity and currency markets. In stock markets, the REH solution of the model typically sets the equilibrium price, P_t^{RE} , equal to a constant multiple of the current dividend.⁵⁴ Market participants and policy makers have long recognized measures of the historical averages of this ratio and its close correlate, the PE ratio, as benchmarks around which stock prices revolve. As for currency markets, the REH solution of the model sets P_t^{RE} equal to the PPP exchange rate, P_t^{PPP} . Here, too, measures of the PPP exchange rate have a long history dating back to the 1500s as a useful benchmark in gauging the extent of currency fluctuations.⁵⁵ Thus, in the context of currency markets, we can write (30) as⁵⁶

$$P_t = P_t^{\text{PPP}} + c \left(\hat{P}_{t|t+1}^{\text{IK}} - \hat{P}_{t|t+1}^{\text{RE}} \right) \quad (31)$$

In the next section, we use (31) to illustrate that once REH is replaced by

⁵²See Blanchard and Fisher (1989) and references therein for studies that use equation (1) to model stock prices, as well as goods prices. See Frydman and Goldberg (2007, chapter 6) and references therein for studies that rely on this specification for currency prices.

⁵³This way of thinking about asset price swings—that they arise from autonomous movements in the aggregate forecast—also underlies behavioral models of swings. See, for example, the seminal study of Frankel and Froot (1987). However, these models fully prespecify revisions in market participants' forecasts; thus, as with other behavioral models of market outcomes, they presume irrationality on the part of individuals. An open question is whether the Frankel and Froot (1987) model and its extensions (see De Grauwe and Grimaldi, 2006, and references therein) can be re-interpreted in an IKE framework.

⁵⁴This REH solution assumes that the dividend process follows $D_t = D_{t-1}(1 + \mu^D)$, where μ^D is the constant growth rate of dividends. With this specification, and the assumption of a fixed risk-free rate, r , we have $P_t^{\text{RE}} = D_t / (r - \mu^d)$. The REH solution can be expressed more generally in terms of the mean growth rate of dividends and the mean risk-free rate when μ^D and r are allowed to vary over time.

⁵⁵International macroeconomists trace the notion of PPP back to scholars at the University of Salamanca in the fifteenth and sixteenth centuries. See Officer (1976). For formal evidence that exchange rate swings do revolve around PPP levels, see Taylor and Taylor (2004) and references therein.

⁵⁶Although the benchmark levels used in figures 1 and 2 seem to be relevant on historical grounds in both equity and currency markets, establishing their theoretical microfoundations in a world of imperfect knowledge is beyond the scope of this paper.

an IKE representation of forecasting behavior, the model is able to account for the tendency of the exchange rate to undergo long swings.

12 An IKE View of Long Swings

The modern research program in economics models aggregate outcomes by relating them to individual decision-making. The constraints that an IKE model imposes on its representations of individual behavior are not only qualitative, but context-specific. They depend on which aspects of the causal mechanism driving aggregate outcomes an economist seeks to explain.

If the model aims to account for movements in the exchange rate, its qualitative predictions should be consistent with the tendency of this price to undergo swings of uneven duration and magnitude around PPP. This *uneven regularity* at the aggregate level suggests that an economist should look for and formalize qualitative regularities on the individual behavior that are similarly uneven.

The microfoundations of our model of currency swings represent an individuals' forecast of the future exchange rate as

$$\hat{P}_{t|t+1}^i = \beta_t^i Z_t^i \quad (32)$$

This representation shows that there are two key factors that underpin the evolution of an individual's forecast over time: revisions of her forecasting strategy—changes in β_t^i , which could include changes in the composition of the set of causal variables Z_t^i —and movements in the causal variables.⁵⁷ To model currency fluctuations, therefore, we need to look for uneven regularities in these two components.

To illustrate our model of currency swings, we begin with the assumption that the causal variables in the model follow random walks with constant drifts.⁵⁸ This specification enables us to focus much of our attention on revisions of forecasting strategies.

⁵⁷For notational ease, we omit the superscript “IK” on representations on the individual level.

⁵⁸There is much evidence that the usual macroeconomic fundamentals, such as money supply and income levels, are well approximated as unit root processes with drift. See Juselius (2007) and references therein. Although most empirical researchers model many macroeconomic times series as I(1) processes, Juselius *et al* (2007) and Frydman *et al.* (2008) show that they are also well approximated as near-I(2) processes. Such behavior is also found in Johansen (1997), Kongsted *et al.* (1999), Juselius (2006), and Nielsen and Rahbek (2007).

Economic policy, of course, does change from time to time as new policy makers take charge, economic and social conditions change, or as policy makers' understanding of those conditions evolves. As with market participants' forecasting, policy shifts may also display uneven qualitative regularities that might help in accounting for certain features of currency fluctuations. However, to highlight how IKE representations of forecasting behavior help to account for the uneven nature of currency swings, we maintain the assumption of a fixed policy environment for most of this section.⁵⁹

12.1 Conservatism as a Qualitative Regularity of Uneven Duration

To model revisions of an individual's forecasting strategy, we explore the implications of a well-documented phenomenon that psychologists call "conservatism:" individuals tend to revise the ways that they form their beliefs about uncertain outcomes gradually, relative to some baseline.⁶⁰

While market participants may tend to behave conservatively, conservatism is a regularity that is at best qualitative and uneven.^{61,62} Conservatism should not be expected to characterize individual behavior forever: eventually, the unfolding historical record on market outcomes, changes in the social context, including policy, or the sheer creativity in thinking about the future, may lead a market participant to revise her forecasting strategy in a more substantial, non-conservative way. Conservatism thus offers the possibility of accounting for the kind of price movements we observe in asset markets, in which up-swings are followed by down-swings in ways and at times that do not follow

⁵⁹In section 12.7.1, we show how the relaxation of this assumption can help explain the boundedness of fluctuations in currency markets. A more general IKE model that incorporates qualitative features of policymaking is beyond the scope of this paper. For a first step in this direction, see Frydman and Goldberg (2004).

⁶⁰See Edwards (1968) and Shleifer (2000).

⁶¹This seems to be the case with other empirical findings that behavioral economists have uncovered. For example, they report much evidence that participants in financial markets often rely on technical trading rules in deciding when to take open positions. However, there is also much evidence that the importance individuals place on such strategies varies over time. See Schulmeister (2006, 2008) and references therein. By contrast, we would expect that the regularities that characterize individuals' preferences—for example, the importance of fairness or loss aversion in individual decision making—may be more enduring.

⁶²Barberis, Shleifer, and Vishny (1998) also appeal to conservatism in modeling an individual's forecasting behavior. But, to generate sharp predictions, they formulate conservative behavior with a fixed rule that presumes that individuals under-react to earnings announcements in exactly the same way at every point in time.

any fully prespecified rule.

How one would formalize conservatism depends on the context. In general, it requires a specification of both the formation of beliefs and the baseline against which revisions of those beliefs are judged.⁶³

In the context of our model, the way in which an individual forms her beliefs—her forecast—is represented by $\beta_t^i Z_t^i$. Our baseline is defined by the change in an individual’s forecast that would occur if she decided to leave her forecasting strategy unchanged, which we call the “status quo” change. Conservatism is then defined in terms of the change of $\widehat{P}_{t|t+1}^i$ that arises from revisions of an individual’s forecasting strategy, relative to the change that is associated with the status-quo.

Given the representation in (32), and the assumption that the causal variables follow random walks with deterministic drifts, the total change in an individual’s forecast is

$$\widehat{P}_{t|t+1}^i - \widehat{P}_{t-1|t}^i = \mathcal{T}\widehat{P}_{t|t+1}^i + \epsilon_t^i \tag{33}$$

where

$$\mathcal{D}\widehat{P}_{t|t+1}^i = \Delta\beta_t^i Z_t^i + \beta_{t-1}^i \mu^{Z^i} \tag{34}$$

$\mathcal{T}\widehat{P}_{t|t+1}^i$ is the trend change in the individual’s forecast between $t-1$ and t , ϵ_t^i is a vector of mean-zero, i.i.d. errors that stem from the Z_t^i process, and the “ \mathcal{T} ” notation underscores the fact that with IKE, a change in $\widehat{P}_{t|t+1}^i$ may result from a change in the structure of the representation and not merely from updating due to new information on the causal variables.⁶⁴ We define our baseline in terms of the trend change in $\widehat{P}_{t|t+1}^i$ that would occur if an individual keeps her forecasting strategy unaltered and she updates her forecast solely because of drifts in causal variables. The expression in (34) shows that this baseline “drift” is given by $\beta_{t-1}^i \mu^{Z^i}$.⁶⁵ The expression shows that $\mathcal{T}\widehat{P}_{t|t+1}^i$ also depends on how she revises her forecasting strategy, $\Delta\beta_t^i$.⁶⁶

⁶³In Edwards’ experiments, conservatism was defined as updating that is too slow relative to a baseline defined by the updating associated with Bayes’ rule.

⁶⁴The \mathcal{T} operator serves a role that is analogous to the total differential, which in general is not well defined when both the arguments and the structure of the representation change.

⁶⁵In Frydman and Goldberg (2007), the baseline change is defined slightly differently to account for the fact that the PPP exchange rate moves over time.

⁶⁶At every point in time, the vector Z_t^i represents all possible casual factors that an individual might use in forming her forecast. As such, the representation of change in (33) allows for the composition of the causal variables to change. For example, if an individual is presumed to use a particular variable Z_j to form her forecast at t but not at $t-1$, then $\beta_{j,t-1}^i = 0$ and $\Delta\beta_{j,t}^i = \beta_{j,t}^i$.

Our IKE model of currency swings under flexible goods prices makes use of two conservative conditions, which represent how individuals may revise their forecasting strategies. One of these conditions restricts revisions of β_t^i so that their impact on the level of $\widehat{P}_{t|t+1}^i$ is smaller in size than the baseline drift, that is,

$$|\Delta\beta_t^i Z_t^i| < \delta_t \quad (35)$$

where $|\cdot|$ denotes an absolute value and $\delta_t = |\beta_{t-1}^i \mu^{Z^i}|$ is the magnitude of the baseline drift.

A revision of β_t^i at a point in time, say $t = \tau$, impacts not only the change in an individual's forecast between $\tau - 1$ and τ , but also the baseline change between τ and $\tau + 1$, that is, $\widehat{\beta}_\tau^i \mu^{Z^i}$. The second conservative condition restricts revisions of β_t^i so that the baseline drifts in two consecutive periods have the same sign. Given that $\beta_t^i \mu^{Z^i} = \beta_{t-1}^i \mu^{Z^i} + \Delta\beta_t^i \mu^{Z^i}$, the following condition ensures that if $\beta_{t-1}^i \mu^{Z^i}$ implies an upward or downward movement in $\widehat{P}_{t|t+1}^i$, so will $\beta_t^i \mu^{Z^i}$:

$$|\Delta\beta_t^i \mu^{Z^i}| < \delta_t \quad (36)$$

The constraints (35) and (36) embody the idea that when an individual decides to revise her strategy, she is reluctant to do so in ways that would alter the level or baseline drift of her forecast too much from what would be associated with the status quo. The baseline drift in our formulation of conservatism is the change that is produced in standard models, which arises from the mere updating due to movements of causal variables.

Economists sometimes recognize that changes in causal variables may lead to revisions of individuals' forecasting strategies. But, when they do, they rely on pre-existing rules, like Bayes' formula. By contrast, our IKE formulation recognizes that individuals do not endlessly obey pre-existing rules in deciding on when and how to alter their forecasting strategies. Indeed, the decision to revise one's forecasting strategy depends on many factors, including prior forecasting success, economic and political developments, emotions, or, as we will suggest shortly, the size of the departure of the exchange rate from PPP.

The conservative restrictions in (35) and (36) leave room for non-rule-based revisions by restricting neither the causal variables that may enter the representation in (32), nor how exactly these variables might matter. Moreover, they do not constrain the trend change in an individual's forecast to be small in magnitude, only that this change is conservative relative to the status quo, in which the causal variables drive change. Consequently, if changes in the causal variables between two points in time were large, the change in $\widehat{P}_{t|t+1}^i$ could also be large.

Because we formalize conservatism with qualitative conditions, our representation of forecasting behavior is consistent with myriad possible changes in β_t^i and the composition of Z_{t-1}^i . Our IKE representation, therefore, implies myriad conditional probability distributions for the equilibrium price at t —one for each possible value of β_t^i and set of causal variables—conditional on any one of the distributions at time $t - 1$.⁶⁷ In this way, the representation in (33)-(36) recognizes the role of both forecast revisions that do not follow pre-existing rules and new information for explaining outcomes.

12.2 Uneven Swings in an Individual's Forecast

Although only qualitative, the conservative conditions in (35) and (36) place sufficient structure on the analysis: any time period in which revisions of an individual's forecasting strategy are conservative, and the drifts in the causal variables remain unchanged, will be characterized by movements of $\widehat{P}_{t|t+1}^i$ that are, on average, in one direction. To see this, suppose that the base-line drift driving an individual's forecast is initially positive between $t = \tau - 1$ and τ , that is, $\beta_{\tau-1}^i \mu^{Z^i} > 0$. This positive drift implies that if the individual decides to leave her forecasting strategy unchanged at τ , so that $\Delta\beta_\tau^i = 0$, her forecast will tend to rise between $\tau - 1$ and τ . With imperfect knowledge, however, an individual revises her strategy at least intermittently. Revisions can either reinforce or impede the positive change in $\widehat{P}_{t|t+1}^i$ that is due to the movements of the causal variables. But, if revisions satisfy the constraint in (35), their impact on $\widehat{P}_{t|t+1}^i$ will be smaller than that of the baseline drift and $\widehat{P}_{t|t+1}^i$ will tend to move up between $\tau - 1$ and τ .

The tendency of $\widehat{P}_{t|t+1}^i$ to rise may persist between τ and $\tau + 1$. The matter depends partly on whether the baseline drift behind $\widehat{P}_{t|t+1}^i$ remains positive, that is, whether $\beta_\tau^i \mu^{Z^i} > 0$. But, if the revision of β_τ^i satisfies the constraint in (36), this will be the case. Thus, if an individual revises her forecasting strategy at time $\tau + 1$ and this revision satisfies the constraint in (35), the tendency of $\widehat{P}_{t|t+1}^i$ to rise will endure between τ and $\tau + 1$. Moreover, it is clear that this tendency for $\widehat{P}_{t|t+1}^i$ to rise will persist as long as the revisions of individual i 's forecasting strategy are conservative, as defined by the conditions in (35) and (36).

It does not seem plausible that a market participant would revise her forecasting strategy in conservative ways endlessly. Indeed, for an upward swing

⁶⁷This contrasts sharply with the fully predetermined representation of forecasting behavior in section 7, which implies just one probability distribution at t .

in $\widehat{P}_{t|t+1}^i$ to end, the individual would have to revise her forecasting strategy in a non-conservative and non-reinforcing way at some point in time. Such a revision could be associated with a short-lived reversal in $\widehat{P}_{t|t+1}^i$ or one that lasts for many periods. A sustained reversal in $\widehat{P}_{t|t+1}^i$ back towards parity would begin at some point in time, say $\tau + T$, if 1) the revision of β_t^i at $\tau + T$ leads to a negative baseline drift; and 2) revisions of β_t^i are once again conservative for an extended period of time beyond $\tau + T$.

12.3 Bulls, Bears, and Long Swings

The qualitative prediction that an individual's forecast undergoes a swing during an extended period of time is conditional on the way she revises her forecasting strategy and on how the causal variables move over time. Consequently, the predictions on the aggregate level that might follow from such representations on the individual level are also conditional on how market participants' revise their forecasting strategies and on how the causal variables develop over time.

If we were to follow the vast majority of our colleagues and assume that all market participants share the same forecast, then movements of an individual's forecast would be tantamount to movements of the aggregate forecast. However, imperfect knowledge in real-world markets gives rise to a diversity of forecasting strategies across market participants. In asset markets, this diversity takes on a striking form: at each point in time, some participants forecast a rise in the asset price while others forecast a fall.

Recognizing this kind of diversity is crucial for explaining outcomes in currency markets. However, as we discuss in section 14, to do so without the presumption of irrationality, an economist must not only stop short of fully prespecifying change, but the qualitative constraints that he imposes on the revisions of forecasting strategies cannot apply endlessly.

Market bulls hold long positions in foreign exchange because they expect the exchange rate to rise over the holding period, while bears hold short positions because they expect the exchange rate to fall. To represent the forecasting behavior of bulls and bears, we aggregate over these groups using wealth shares;

$$\widehat{R}_{t|t+1}^L = \widehat{P}_{t|t+1}^L - P_t > 0 \quad (37)$$

$$\widehat{R}_{t|t+1}^S = P_t - \widehat{P}_{t|t+1}^S > 0 \quad (38)$$

where $\widehat{P}_{t|t+1}^L = \beta_t^L Z_t^L$ and $\widehat{P}_{t|t+1}^S = \beta_t^S Z_t^S$ represent aggregates of the exchange rate forecasts of the bulls and bears, respectively, and $\widehat{R}_{t|t+1}^L$ and $\widehat{R}_{t|t+1}^S$ repre-

sent aggregates of bulls' and bears' expected returns on holding long and short positions, respectively.⁶⁸ In the aggregate, we have,

$$\widehat{P}_{t|t+1}^{\text{IK}} = \frac{1}{2} \left(\widehat{P}_{t|t+1}^{\text{L}} + \widehat{P}_{t|t+1}^{\text{S}} \right) = \beta_t^{\text{IK}} Z_t \quad (39)$$

where Z_t^{IK} represents the union of causal variables that participants use in forming their forecasts and β_t^{IK} denotes weighted averages of the parameters that they attach to these variables.⁶⁹

Equation (31) shows that exchange rate swings arise because the aggregate of individuals' forecasts, $\widehat{P}_{t|t+1}^{\text{IK}}$, undergoes a swing. The specification in equation (39) reveals that a swing in $\widehat{P}_{t|t+1}^{\text{IK}}$ in one direction or the other depends not on the constellation of participants' beliefs about whether the exchange rate will rise or fall, but on how those beliefs change over time. So if $\widehat{P}_{t|t+1}^{\text{IK}}$ were to rise, on average, over an extended period of time, then the exchange rate would also tend to undergo an upswing even though there are bears in the market who bet on the opposite movement. Such change could come about because both bulls and bears steadily raise their exchange rate forecasts or because the forecasts of only one side of the market increase.⁷⁰ Whatever the case, however, the exchange rate would tend to rise as long as $\widehat{P}_{t|t+1}^{\text{IK}}$ tended to rise. If this movement were toward PPP initially, and the period of a rising $\widehat{P}_{t|t+1}^{\text{IK}}$ were to endure, eventually the exchange rate would shoot through this benchmark and continue trending from the other side. It is also clear that this swing away from PPP would end if the swing in $\widehat{P}_{t|t+1}^{\text{IK}}$ were to end.

It is not difficult to see from our analysis on the individual level how conservatism would lead to such a swing in $\widehat{P}_{t|t+1}^{\text{IK}}$ in our model: the aggregate $\widehat{P}_{t|t+1}^{\text{IK}}$ is just a weighted sum of the individual forecasts. If we assume that each market participant revises her forecasting strategy in conservative ways over an extended period of time, then each component of the aggregate will involve a trend that, although time-varying, does not change direction. Consequently, the trend in the aggregate forecast will also involve an unchanging sign and thus imply a tendency for $\widehat{P}_{t|t+1}^{\text{IK}}$ to move in one direction over time.

⁶⁸Positive realizations of $P_{t+1} - P_t$ imply profits on long positions and losses on short positions. We thus define the return on a short position as $R_{t+1}^{\text{S}} = P_t - P_{t+1}$. The definitions of the returns on open positions abstracts from the cost of capital and thus from interest rates for convenience only.

⁶⁹In our analysis, we assume that the wealth shares of the groups of bulls and bears are exogenous and constant, so we set them equal to a half.

⁷⁰The rise in the forecasts of some individuals might even cause them to switch from a bear to a bull.

The assumption that all market participants behave conservatively is strong. At each point in time we would expect that some individuals would behave conservatively while others would not. Whether the aggregate forecast tends to move in one direction over a period of time would then depend on the relative weight of the individuals in the market who behave conservatively. A swing would arise in the model during those periods in which this weight remained sufficiently high.

12.4 Long Swings and Fundamentals

Many international macroeconomists view the long-swings behavior of exchange rates as evidence that these prices do not depend on macroeconomic fundamentals, such as income levels and interest rates. Their empirical studies, which search for a fully prespecified (mostly fixed) relationship between the exchange rate and macroeconomic fundamentals, appear to confirm this view.⁷¹

Our model of currency swings provides an alternative view. It is clear from our analysis that swings arise in the model even if the forecasting strategies attributed to market participants are assumed to depend solely on macroeconomic fundamentals.⁷² Moreover, to account for currency swings that are uneven in duration and magnitude, we need to allow for revisions of market participants' forecasting strategies. But, equations (31) and (33) show that such behavior is associated with structural change in the relationship between the exchange rate and fundamentals. Thus, even if fundamentals matter for currency movements, we should not expect, as the majority of extant studies do, that a time-invariant model involving a fixed set of fundamentals would adequately represent the causal relationship for periods of time as long as decades. In fact, formal empirical analysis reveals not only that this relationship has been unstable during the modern era of floating currencies, but that the set of fundamentals that matters has changed from one sub-period to another.⁷³

Our theoretical and empirical findings suggest that long swings in exchange rates away from parity occur not because market participants ignore fundamentals in forming their forecasts, but because knowledge is imperfect and fundamentals tend to drift in particular directions for extended periods of

⁷¹For a review of this literature, see Frankel and Rose (1995). For a discussion, see section 15.1.

⁷²This is true even if Z_t includes only those fundamentals that drive the REH forecast, that is, if we set $Z_t = X_t$.

⁷³See Goldberg and Frydman (1996) and Frydman and Goldberg (2007, chapter 15).

time.

12.5 Conditional Predictions of Change

As with the model's predictions on the individual level, its predictions about the equilibrium exchange rate are conditional on how individuals revise their forecasting strategies and on the movements of the causal variables. The model implies an exchange rate swing during any period of time in which 1) revisions of strategies are conservative; and 2) drifts in the causal variables remain unchanged. The assumption that conservative forecasting behavior does not last forever is crucial for the model's empirical relevance. Indeed, as we have shown, in order to account for the exchange rate swings in figure 1, which are of limited and uneven duration, we need to assume that revisions of forecasting strategies are, at unpredictable points in time, non-conservative and non-reinforcing. A change in policy may also trigger a sustained reversal in the exchange rate (see section 12.6). However, in any case, we must assume that the constraints on change do not apply endlessly if we want to explain the uneven nature of exchange rate swings.⁷⁴

Findings in psychology suggest that individuals are often reluctant to revise their decision-making strategies in dramatic ways. And macroeconomic fundamentals often trend in particular directions for years at a time. Conditional on such behavior, our model implies exchange rate swings of significant duration and magnitude. However, we cannot prespecify exactly when forecasting behavior will be conservative or when the drifts in the casual variables will be constant. Hence, we cannot prespecify exactly when exchange rate swings will begin or end. As we have noted, this feature of the model is crucial for avoiding internal inconsistency and the presumption of gross irrationality on the part of market participants.

12.6 Bounded Instability and the Role of Benchmarks

Beyond its conditional prediction of uneven currency swings, our IKE model implies that swings away from PPP are ultimately bounded. As the gap from PPP grows, either market participants eventually cease to be conservative, owing to the way they assess the risk of capital losses on holding open positions, or policy officials alter the course of the causal variables.

⁷⁴In fact, with no change in structure of any kind, or with individuals who behave conservatively endlessly, the model implies, counterfactually, that the exchange rate will undergo an unbounded swing away from PPP. See chapter 14 of our book.

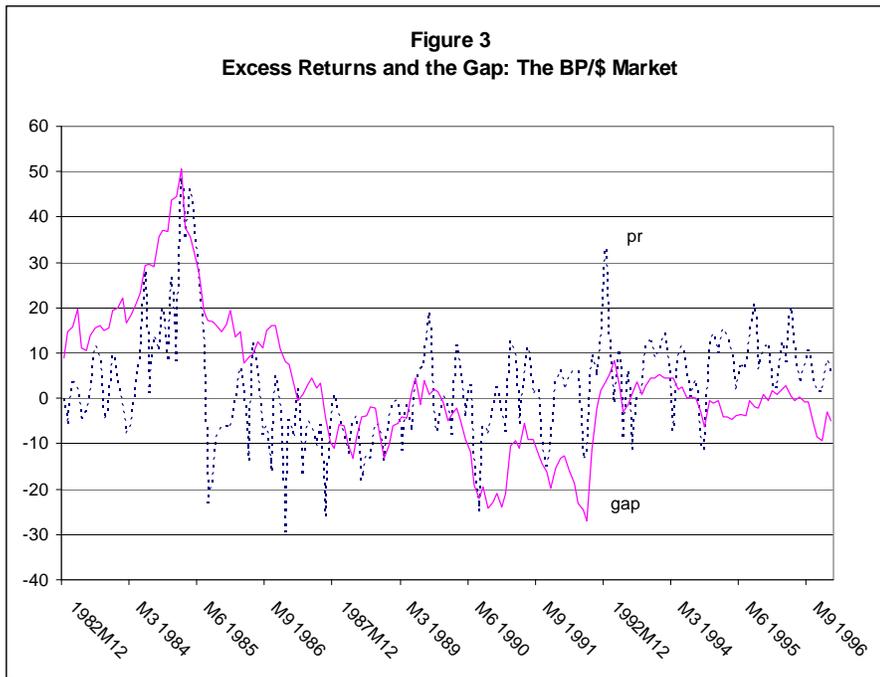
Our representation of risk replaces the usual assumptions of risk aversion and expected utility theory with what we call *endogenous prospect theory*. This alternative representation of preferences under uncertainty implies that an individual's degree of loss aversion increases as the size of her open position in the market increases.⁷⁵ Frydman and Goldberg (2007) show that with this specification, both bulls and bears expect to earn a positive return – a premium – for holding open positions in the market, which depends on one's forecast of the potential loss from speculation. To model an individual's speculative decision, therefore, and thus the movement of the exchange rate, we must represent how revisions of her forecasting strategy alter both her forecast of the next period's exchange rate, $\hat{P}_{t|t+1}^i$, and her forecast of the potential loss from holding an open position, which we denote by $\hat{l}_{t|t+1}^i$.

To this end, we make use of the historical record on the co-movement of two variables: the aggregate premium—expected return—on foreign exchange, which we denote by $\hat{pr}_{t|t+1}^{\text{IK}}$, and the gap between the aggregate forecast and the PPP exchange rate, which we denote by $\widehat{gap}_t^{\text{IK}} = \hat{P}_{t|t+1}^{\text{IK}} - \hat{P}_t^{\text{PPP}}$. In figure 3, we plot $\hat{pr}_{t|t+1}^{\text{IK}}$ and $\widehat{gap}_t^{\text{IK}}$ for the British pound-U.S. dollar (BP/\$) market.⁷⁶ The figure clearly suggests that $\hat{pr}_{t|t+1}^{\text{IK}}$ tends to move positively over time with $\widehat{gap}_t^{\text{IK}}$.⁷⁷ The results of regression analysis also indicate a positive relationship between $\hat{pr}_{t|t+1}^{\text{IK}}$ and $\widehat{gap}_t^{\text{IK}}$, although they show that the quantitative relation-

⁷⁵An individual is loss averse if her disutility from losses is greater than her utility from gains of the same magnitude. Endogenous prospect theory provides a way to represent the experimental findings of Kahneman and Tversky (1979) and others in a world of imperfect knowledge.

⁷⁶We use survey data on exchange rate expectations to measure $\hat{pr}_{t|t+1}^{\text{IK}}$ and the *Big Mac* PPP exchange rate as in figure 1. The survey data are from Money Market Services International (MMSI), which entail median responses from market participants concerning their four-week ahead point forecasts of the exchange rate. For more details concerning the time plots in figure 2, see Frydman and Goldberg (2007, chapter 12). Other studies that have used survey data from MMSI include Frankel and Froot (1987) and Froot and Frankel (1989).

⁷⁷Time plots for the DM/\$ and Japanese yen/\$ markets show a similar pattern. We first made use of the positive relationship between $\hat{pr}_{t|t+1}^{\text{IK}}$ and $\widehat{gap}_t^{\text{IK}}$ in Frydman and Goldberg (2003) to explain the forward-discount puzzle.



ship between these aggregates varies over the sample period.⁷⁸

12.7 Benchmark Levels and Revisions of Forecasting Strategies

Our representations of forecasting behavior imply that when an individual revises her strategy, she alters her forecasts of the future exchange rate and potential loss. She may be conservative in how she changes both of these forecasts. However, because conservatism does not involve any restrictions on the causal mechanism—the composition of the causal factors and how they influence market outcomes—it is not sufficient to account for the relationship between the aggregate premium and the gap. Thus, in formulating the microfoundations of a model that might explain the regularity between these variables, we must look for additional regularities concerning forecasting behavior on the individual level.

To this end, we build on Keynes (1936) and relate revisions of $\hat{l}_{t|t+1}^i$ to the gap between an individual's forecast of the exchange rate and her measure

⁷⁸See Frydman and Goldberg (2007, chapter 12). Our empirical analysis relies on recursive methods that enable us to test for structural change without prespecifying its timing or nature. Moreover, we look for—and find—some evidence that the market premium also depends on cumulative current account imbalances, where, again, this relationship is only qualitative.

of the benchmark exchange rate. Keynes argued that, while asset prices have a tendency to move persistently away from benchmark levels for protracted periods, they eventually undergo, at unpredictable moments, sustained countermovements back to these levels. He recognized that market participants are also aware of this feature of the social context and use it in their attempts to forecast future market outcomes. In discussing why an individual might hold cash rather than interest-bearing bonds, Keynes observed that “what matters is not the absolute level of r but the degree of its divergence from what is considered a fairly safe level of r , having regard to those calculations of probability which are being relied on” (Keynes, 1936, p.201).

A benchmark level is, of course, specific to each asset market. Every individual arrives at her own determination of the benchmark value and so, in general, these assessments will differ across individuals. How individuals come to decide on a benchmark level is an open question. Keynes suggests in his discussion that conventions and the historical record play an important role. For example, as we have already mentioned, PPP has long played a role in how market participants and policy makers assess the misalignment of exchange rates, and its role as a benchmark is reasonable in view of the empirical record in many currency markets. This is also the case with historical PD and PE ratios in equity markets.

Keynes’s discussion of the importance of benchmark levels as anchors for exchange rate fluctuations suggests that a market participant’s forecast of the potential loss from speculation depends on her evaluation of the gap between the asset price and its benchmark level. For example, we would expect that as the asset price rises further above the perceived benchmark level, bulls (bears) would become more concerned (more confident) about a reversal and thus greater (smaller) capital losses.⁷⁹

As with conservatism, we would not expect that these regularities for bulls and bears would conform to any fully prespecified rule. We thus rely on qualitative conditions to represent them, which we call gap restrictions.⁸⁰ These representations of forecasting behavior are easily illustrated.

⁷⁹As Keynes puts it, “Unless reasons are believed to exist why future experience will be very different from past experience, a ...rate of interest [much lower than the safe rate], leaves more to fear than to hope, and offers, at the same time, a running yield which is only sufficient to offset a very small measure of fear [of capital loss]” (Keynes, 1936, p.202).

⁸⁰We also make use of additional qualitative conditions that appeal to theoretical considerations to model revisions in $\hat{l}_{t|t+1}^i$. Economic theory suggests that market participants might also consider current account imbalances in forecasting potential losses. See Frydman and Goldberg (2007, chapter 12).

An individual's expected loss from speculation can be written as,

$$\widehat{l}_{t|t+1}^{i,L} = E_t^i[R_{t+1}^L < 0|Z_t^i] < 0$$

for a bull and,

$$\widehat{l}_{t|t+1}^{i,S} = E_t^i[R_{t+1}^S < 0|Z_t^i] < 0 \tag{40}$$

for a bear, where $R_{t+1}^L = P_{t+1} - P_t$ and $R_{t+1}^S = P_t - P_{t+1}$ are the returns on long and short positions on foreign exchange, respectively, and $E_t^i[R_{t+1}^L < 0|Z_t^i]$ and $E_t^i[R_{t+1}^S > 0|Z_t^i]$ denote the expected value of the realizations on R_{t+1}^L and R_{t+1}^S that imply a loss for a bull and bear, respectively, conditional on an individual's forecasting strategy and information set.

The gap conditions for an individual bull and bear can be written as follows

$$\frac{\mathcal{T}\widehat{l}_{t|t+1}^{i,L}}{\mathcal{T}\widehat{gap}_t^{i,L}} < 0 \quad \text{and} \quad \frac{\mathcal{T}\widehat{l}_{t|t+1}^{i,S}}{\mathcal{T}\widehat{gap}_t^{i,S}} > 0 \tag{41}$$

where the \mathcal{T} operator, as before, denotes a total change, $\widehat{gap}_t^i = \widehat{P}_{t|t+1}^i - \widehat{P}_t^{i,PPP}$, and $\widehat{P}_t^{i,PPP}$ denotes an individual's assessment at t of the PPP exchange rate.⁸¹ These qualitative conditions constrain our representation of how an individual revises her forecasting strategy by restricting all post-change conditional probability distributions to imply a gap effect: distributions that imply a higher \widehat{gap}_t^i also imply an expectation of greater (smaller) potential losses if the individual is a bull (bear).⁸²

12.7.1 Self-Limiting Long Swings

To see how our IKE model with the gap conditions in (41) implies that exchange rate swings away from PPP will eventually end, consider an extended period of time in which market participants revise their forecasting strategies in conservative ways. Suppose that initially, forecasting strategies and the drifts in the causal variables imply a positive baseline drift in $\widehat{P}_{t|t+1}^{i,K}$, that is, $\beta_{\tau-1}^{i,K}\mu^Z > 0$. During this period of time, then, $\widehat{P}_{t|t+1}^{i,K}$ and the exchange rate,

⁸¹The gap could also be defined in terms of the time- t exchange rate, rather than the forecast of the future exchange rate, or some weighted average of the two without affecting the conclusions of our analysis. See Frydman and Goldberg (2003).

⁸²Because $\widehat{l}_{t|t+1}^{i,L}$ and $\widehat{l}_{t|t+1}^{i,S}$ are both negative, greater (smaller) losses imply a fall (rise) in these magnitudes. Hence, the less-than and greater-than inequalities in formulating the gap conditions for a bull and bear, respectively.

P_t , will tend to rise. To keep our example simple, we focus only on the role of bulls and assume that the rise in $\widehat{P}_{t|t+1}^{\text{IK}}$ is associated with a rise in $\widehat{P}_{t|t+1}^{\text{L}}$.⁸³

While conservative revisions of strategies and movements in causal variables lead bulls to raise their forecasts and bid the exchange rate further above PPP, our IKE model of risk indicates that they simultaneously become more concerned about a sustained counter-movement—that is, about capital losses. This leads them to raise the premiums that they require to increase their long positions. According to our model, if the swing away from PPP were to continue, a threshold would eventually be reached at which bulls would become so concerned about a reversal that they would no longer revise their forecasting strategies in conservative ways. At that point, they would either reduce their long positions or abandon them altogether, which would precipitate a reversal in the exchange rate.

PPP matters because market participants have come to rely on this benchmark in their attempts to forecast potential losses and assess the riskiness of holding open positions. When, exactly, the gap from PPP, and thus the potential loss, is perceived by bulls to be too large for them to continue bidding up the exchange rate depends on many factors, including economic, political, and policy considerations that no one can fully prespecify. Thus, no one can fully prespecify when long swings away from PPP will eventually end.

Policy makers also use PPP as a benchmark level in setting economic policy and their actions also play an important role in keeping exchange rate swings bounded. We have so far assumed in our discussion of long swings that trends in macroeconomic fundamentals remain fixed. However, the empirical record shows that policy officials sometimes worry about large departures in the exchange rate from PPP,⁸⁴ and alter policy to engender a reversal.⁸⁵

As with revisions of forecasting strategies, we would not expect that such a regularity would follow any fully prespecified rule. This reasoning leads us to assume that beyond some threshold, which we do not prespecify, policy makers respond to large departures from PPP by altering policy to limit the

⁸³The focus on bulls enables us to avoid distributional issues. But, of course, the bears also play a role in keeping the swing away from PPP bounded. See Frydman and Goldberg (2007).

⁸⁴Large misalignments pose challenges for firms that are engaged in international business by leading to changes in competitiveness. These effects, in turn, lead to calls for protectionist measures, which may reduce the benefits from international trade for economic activity.

⁸⁵Examples of such behavior include the coordinated interventions by central banks and the changes in monetary and fiscal policies that were aimed at bringing down U.S. dollar rates in 1985 and yen rates in 1995, as well as the interventions by the U.S. Federal Reserve and the European Central Bank to stem the fall of the dollar in 2007 and the first half of 2008.

misalignment. The fact that many of the major reversals in currency markets are proximate to major changes in policy suggests that the behavior of policy makers plays an important role in keeping long swings in the exchange rate bounded.^{86,87}

13 The Market Premium and the Aggregate Gap

Beyond its implications for keeping exchange rate swings away from PPP bounded, our IKE representation of bulls' and bears' expected losses implies a new specification for the causal mechanism underpinning the equilibrium premium on foreign exchange, $\widehat{pr}_{t|t+1}^{\text{IK}}$, that is able to account for the qualitative regularity evident in figure 2.

Endogenous prospect theory implies that the equilibrium premium on foreign exchange depends on an aggregate of the returns that bulls expect to earn from holding long positions relative to the returns that bears expect from their short positions:

$$\widehat{pr}_{t|t+1}^{\text{IK}} = \frac{1}{2} (\widehat{up}_{t|t+1}^{\text{L}} - \widehat{up}_{t|t+1}^{\text{S}}) \quad (42)$$

where, echoing Knight's (1921) distinction between uncertainty and risk, we refer to the expected returns of the bulls and bears, $\widehat{up}_{t|t+1}^{\text{L}} > 0$ and $\widehat{up}_{t|t+1}^{\text{S}} > 0$, respectively, as uncertainty premiums.⁸⁸ We show that if forecasting behavior is consistent with the gap conditions in (41), the market premium will move positively with the aggregate gap, $\widehat{gap}_t^{\text{IK}}$, over time. The intuition follows from the connection between individuals' premiums and their forecasts of the potential losses; as $\widehat{gap}_t^{\text{IK}}$ rises, foreign exchange bulls (bears) become more (less) concerned about the potential losses from holding open positions in the market. Thus, in equilibrium, the uncertainty premiums of the bulls (bears)

⁸⁶For example, the major reversals in U.S. dollar rates in late 1979 and early 1985 were associated with the arrival of Paul Volcker and James Baker, respectively, both of whom quickly engineered major changes in policy. An example of a connection between policy and major reversals in other asset markets is provided by the downturn in U.S. equity markets that began in August 2000, which came on the heels of the Federal Reserve's decision to raise the federal funds rate from 4.74 percent in July 1999 to 6.5 percent in May 2000.

⁸⁷Beyond the policy channel, departures in the exchange rate from PPP influence trends in macroeconomic fundamentals endogenously in ways that also keep exchange rate swings bounded. For example, swings in exchange rates eventually lead to changes in current account imbalances and economic growth that would tend to limit such swings.

⁸⁸The market premium also depends positively on the international financial position of the domestic country. See Frydman and Goldberg (2007, chapter 11). To keep the discussion simple, we ignore this feature of the model.

rise (fall). Both of these movements lead to an increase in the equilibrium premium.⁸⁹

One would not expect that the influence of the aggregate gap on the market premium would be the same in every time period. Indeed, we might expect that this effect would be small (large) when the size of the $\widehat{gap}_t^{\text{IK}}$ was small (large).⁹⁰ Nevertheless, our model implies that, although the quantitative relationship between the market premium and the gap varies over time, it does so in ways that preserve the qualitative relationship between these variables: a rise (fall) in $\widehat{gap}_t^{\text{IK}}$ is associated with a rise (fall) in $\widehat{pr}_{t|t+1}^{\text{IK}}$.

Beyond its ability to account for the positive relationship between the market premium and the aggregate gap that we observe in the data, our IKE model is able to explain a feature of the empirical record that extant approaches have found extremely puzzling: the frequent switches in the algebraic sign of $\widehat{pr}_{t|t+1}^{\text{IK}}$ that are apparent in figure 1 and in other currency markets. The expression for the market premium in (42) shows that the algebraic sign of $\widehat{pr}_{t|t+1}^{\text{IK}}$ depends on the size of the bulls' uncertainty premium relative to the bears'. When this relative magnitude changes sign, so too does the sign of the market premium.⁹¹ One of the implications of our model is that the frequency of sign reversals in $\widehat{pr}_{t|t+1}^{\text{IK}}$ declines when the size of the aggregate gap is "large." We show that this is, indeed, the case in the BP, German mark, and Japanese yen markets using thresholds for $\widehat{gap}_t^{\text{IK}}$ of 5, 10, and 20 percent. This result makes clear that allowing for the presence of bulls and bears in the market is crucial to accounting for the time paths of the premium in currency markets.

13.1 Diversity of Forecasting Strategies and the Market Premium

The important role played by the forecasts of bulls and bears in driving the time-path of the premium points to the key reason why REH models have had such difficulties explaining risk in financial markets. Although one may be able to represent the behavior of bulls and bears by assuming some special

⁸⁹There are some distributional issues in deriving this result that we address in Frydman and Goldberg (2007, chapter 12).

⁹⁰We report evidence in Frydman and Goldberg (2007, chapter 12) that supports this conjecture of a non-linear relationship.

⁹¹In the case of a non-zero international financial position, the algebraic sign of $\widehat{pr}_{t|t+1}$ still depends on the relative magnitude $\widehat{up}_{t|t+1}^{\text{L}} - \widehat{up}_{t|t+1}^{\text{S}}$. It is this term, which is absent in conventional models, regardless of whether they allow for bulls and bears, that enables us to account for sign reversals in the premium. See Frydman and Goldberg (2007, chapter 11).

informational asymmetries and a single REH forecasting strategy, in general, the distinction between the two groups' forecasts stems from differences in their forecasting strategies. Moreover, in the context of our model of the premium, the individual gap restrictions in (41) necessarily require that bulls and bears follow diverse forecasting strategies.

To see this, assume that individuals do make use of the same forecasting strategy for the next period's return R_{t+1} and write the conditional mean of the distribution representing this strategy as

$$E_t[R_{t+1}|Z_t] = \widehat{R}_{t|t+1}^+ + \widehat{R}_{t|t+1}^- \quad (43)$$

where $\widehat{R}_{t|t+1}^+ = E_t[R_{t+1} > 0|Z_t] > 0$ and $\widehat{R}_{t|t+1}^- = E_t[R_{t+1} < 0|Z_t] < 0$ denote the expected values of the positive and negative realizations of R_{t+1} , respectively. Furthermore, suppose that a bull and a bear both revise upward their forecast of the future exchange rate between $t - 1$ and t with no change in their estimates of the PPP exchange rate. The increases in $\widehat{P}_{t|t+1}^{i,L}$ and $\widehat{P}_{t|t+1}^{i,S}$ are then tantamount to upward revisions of $\widehat{gap}_t^{i,L}$ and $\widehat{gap}_t^{i,S}$, and, because individuals take P_t as given, a rise in $\widehat{R}_{t+1}^{i,L}$ and a fall in $\widehat{R}_{t+1}^{i,S}$.⁹²

Consider the implications of these revisions for the bull. The gap conditions in (41) imply that the rise in $\widehat{gap}_t^{i,L}$ is associated with a rise in $-\widehat{l}_{t|t+1}^{i,L} = -\widehat{R}_{t|t+1}^{i,-}$. However, in order for both $\widehat{R}_{t+1}^{i,L}$ and $-\widehat{R}_{t|t+1}^{i,-}$ to rise, $\widehat{R}_{t+1}^{i,+}$ must rise, too. Now consider the bear. The gap conditions in (41) imply that the rise in $\widehat{gap}_t^{i,S}$ is associated with a fall in $-\widehat{l}_{t|t+1}^{i,S} = \widehat{R}_{t|t+1}^{i,+}$. However, if the bull and the bear are assumed to follow the same forecasting strategy at $t - 1$, then this common strategy can imply only an increase or a decrease in $\widehat{R}_{t|t+1}^{i,+}$, but not both. Thus, the gap conditions in (41), which specify part of the microfoundations of our model of the market premium, require that bulls and bears follow diverse forecasting strategies at every point in time.

14 How Recognizing the Limits to Knowledge Avoids Internal Inconsistency

We have sketched how our microfounded IKE model of the currency market accounts for three regularities on the aggregate level: long swings in exchange rates of uneven duration and magnitude, a positive relationship between the market premium and the aggregate gap, and the market premium's tendency to undergo sign reversals less frequently when the size of the aggregate gap

⁹²For bears, $\widehat{R}_{t+1}^{i,S} = P_t - \widehat{P}_{t|t+1}^{i,S} > 0$, so a rise in $\widehat{P}_{t|t+1}^{i,S}$ implies a fall in $\widehat{R}_{t+1}^{i,S}$.

is large. Allowing for the presence of bulls and bears, and diverse forecasting strategies among them, played a crucial role in our model of the premium.

In section ??, we showed that if a fully predetermined model allows for diversity of forecasting strategies, its multiple probability distributions representing these strategies are necessarily inconsistent with the single, overarching distribution—sharp prediction—that the model generates on the aggregate level. Following Lucas, we argued that such inconsistent models are the “wrong theory.”

We also showed that in order to avoid inconsistency, an economist must stop short of fully prespecifying the microfoundations of his model, in particular, revisions of forecasting strategies. Because IKE models do so, they imply myriad conditional probability distributions for P_{t+1} on the aggregate level at every point in time. But, although acknowledging the limits to knowledge is necessary to avoid internally inconsistent models, it is not sufficient.

14.1 The Gap Conditions and Market Premium

Consider the model’s predictions concerning the causal mechanism underpinning the premium on foreign exchange. On the aggregate level, the model implies that $\widehat{p}_{t|t+1}^{\text{IK}}$ and $\widehat{gap}_t^{\text{IK}}$ move positively together over time.⁹³ The model also generates myriad of conditional distributions for R_{t+1} , all of which imply a positive relationship between the conditional mean of R_{t+1} and the conditional mean of gap_{t+1} . Thus, to check that our model is not theoretically inconsistent, we must show that the gap conditions in (41), which constrain revisions of forecasting strategies on the individual level, are compatible with a positive relationship between the conditional mean of R_{t+1} and the conditional mean of gap_{t+1} for *both* bulls and bears.

We note that the aggregate regularity between $E_t[R_{t+1}|Z_t]$ and $E_t[gap_{t+1}|Z_t]$ implies no restrictions on how $\widehat{R}_{t|t+1}^+$ and $\widehat{R}_{t|t+1}^-$ may vary separately with $E_t[gap_{t+1}|Z_t]$. Thus, because the gap conditions in (41) constrain how bulls and bears revise their forecasts of the potential loss—which involve, separately, $\widehat{R}_{t|t+1}^-$ for the bulls and $\widehat{R}_{t|t+1}^+$ for the bears—they are compatible with the positive relationship between $E_t[R_{t+1}|Z_t]$ and $E_t[gap_{t+1}|Z_t]$ on the aggregate

⁹³ $\widehat{p}_{t|t+1}^{\text{IK}}$ and $\widehat{gap}_t^{\text{IK}}$ are aggregates of the conditional means of the distributions used to represent the forecasting of individual bulls and bears. Unless one has access to survey data, as we do in figure 2, such aggregates of individuals’ expectations are, in general, not observable; thus, the relationship between them does not, by itself, have any implications for the qualitative properties of probability distributions representing forecasting on the individual level.

level.⁹⁴

14.2 Conservatism and Long Swings

As for the gap conditions, IKE's use of partially predetermined probability distributions enabled us to specify our microfoundations by constraining an aspect of the causal mechanism that differs from the qualitative feature of this mechanism predicted by the aggregate model. This, of course, would not be possible in the context of a fully predetermined model, because these models imply a single overarching distribution of outcomes and thus must disregard diversity—as REH models in fact do—to avoid internal inconsistency.

Conditional on conservative revisions and a fixed policy environment, the aggregate model predicts the direction of change in the exchange rate, either away from or toward the benchmark level. However, it does not have any implications for the way the causal mechanism—the relationship between the exchange rate and a set of causal factors—might change over time.

By contrast, the conservative restrictions constrain the way an individual revises her forecasts of the causal mechanism that underpins the exchange rate. However, they do not restrict in any way an individual's prediction concerning *the direction* of change in the exchange rate. Indeed, the microfoundations of our model allow for both bulls, who forecast appreciation, and bears, who forecast depreciation. Thus, conservative revisions of forecasting strategies on the individual level are compatible with the aggregate model's prediction of long-swings behavior of the exchange rate.

14.3 Bulls and Bears in a Long Swing

The model predicts that, conditional on conservative revisions of forecasting strategies and constant drifts in the causal variables, the exchange rate undergoes protracted swings that revolve around PPP. This prediction appears to conflict with the assumption that the microfoundations of our model allow for bulls and bears: during every up-swing or down-swing, there are market participants who are assumed to bet on a movement of the exchange rate in the opposite direction. The key to avoiding inconsistency, however, lies in

⁹⁴The model constrains a bull's revisions of her forecasting strategy in such a way that if the revised strategy involves a higher (lower) $\widehat{P}_{t|t+1}^{i,L}$ and thus $\widehat{gap}_t^{i,L}$, it will also involve a higher (lower) forecast of the potential loss from speculation, that is, a lower (higher) $\widehat{R}_{t|t+1}^-$ ($\widehat{l}_{t|t+1}^{i,L}$) < 0 . The converse is true for the bears.

recognizing that currency swings are uneven in duration and magnitude: conservative behavior and drifts in the causal variables do not last forever. In the model, non-conservative and non-reinforcing revisions of strategies or changes in policy could occur at any point in time during a protracted swing either up or down. Thus, the model implies that a reversal in the exchange rate could occur at any point in time.

Consequently, the aggregate prediction of a protracted upswing or downswing is not incompatible with the presence of both bulls and bears on the individual level. Because our model does not fully prespecify when exchange rate movements might reverse direction, an individual who remains a bear during an upswing or a bull during a downswing is justified in doing so.

15 The Futile Search for Sharp Predictions

Economists are trained early on to believe that models that do not generate sharp predictions are not worthy of consideration. However, the opposite is true. As John Kay (2007) put it in an article from which the title of this section comes, “the quest for exact knowledge gets in the way of useful knowledge.”

15.1 Lost Fundamentals in Currency Markets

The detrimental effect of the belief that only models that generate sharp predictions are worthy of scientific status is perhaps most evident in the field of financial economics. Consider, for example, how the contemporary approach has impeded economists’ thinking about whether macroeconomic fundamentals matter for currency movements.

There is much anecdotal evidence in the popular media, supported by survey research, that participants in the foreign exchange market pay close attention to fundamental variables in forming their forecasts of future exchange rates. It is obvious, for example, that market participants hang on every word that central bank officials utter, listening for hints of a change in monetary policy. Similarly, two years or so prior to the writing of this paper, market participants clearly responded to announcements of large and growing US current account deficits by selling the dollar. Because individuals’ forecasts drive their behavior in financial markets, we would expect fundamental variables to have considerable influence on exchange-rate fluctuations. Indeed, we showed how currency swings can arise in IKE-based monetary models even if all individuals rely solely on macroeconomic fundamentals in forming their forecasts.

However, in order to build models on the foundation of individual rational

behavior while remaining faithful to the contemporary approach, conventional exchange-rate theorists modeled individual behavior and aggregate outcomes with fully predetermined representations. These conventional models were thought to offer the way to understand how macroeconomic fundamentals and rational behavior affect the exchange rate.

When they failed to find an overarching relationship between the exchange rate and macroeconomic fundamentals, conventional economists concluded that swings in exchange rates away from benchmark levels were unconnected to changes in these fundamentals. Obstfeld and Rogoff (2000) have referred to this “anomalous” finding as the “exchange-rate-disconnect puzzle.” This puzzle led many to presume not only that fundamentals do not matter, but also that some or all market participants behave “irrationally.”

However, in a dynamic world economy, we should not expect to find that a single set of economic fundamentals has mattered in exactly the same way since floating currencies became the norm in the 1970’s. In fact, the exchange-rate-disconnect puzzle disregards empirical evidence, much of it reported by conventional economists themselves, showing that, while macroeconomic fundamentals matter for exchange-rate movements, the causal mechanism that underpins these movements is temporally unstable: not only do the coefficients of empirical models change from one sub-period of floating to another, but the sets of fundamentals that seem to matter for exchange rates also change. Fully predetermined models cannot account for such structural change, the nature and timing of which depends on how market participants revise their forecasting strategies and on unforeseeable changes in the social context.

IKE acknowledges that an overarching model for currency movements is beyond the reach of economic analysis. Moreover, once we allow exchange rate models to undergo structural change at points in time that are not prespecified, statistical analysis reveals that fundamentals do matter for exchange rate movements after all.⁹⁵

15.2 Is the Market Really Grossly Inefficient?

Relying on invariant empirical relationships, many researchers report that future returns in currency markets co-vary negatively with the current value

⁹⁵In chapter 15 of our book, we show that if one allows for structural change, the forecast errors generated by fundamentals-based models are smaller, in mean square error, than the errors generated by the random walk. Thus, we reverse the Meese and Rogoff (1983) conclusion that flipping a coin outperforms structural models.

of the forward premium.⁹⁶ To explain this behavior, conventional economists have constructed exchange rate models in which risk-averse individuals require a positive return, a premium, to hold risky positions in currency markets. It is widely recognized, however, that this research effort has been unsuccessful.⁹⁷

Unable to explain the negative co-variation between the return on foreign exchange and the forward premium that their studies report, economists have reached the startling conclusion that “one can make predictable profits by betting against the forward rate” (Obstfeld and Rogoff, 1996, p. 589). The apparent anomaly that these profits remain unexploited has become one of the major “puzzles” in the international finance literature.

There are several well-known studies in the literature that indicate that the relationship between the return on foreign exchange and the forward premium is temporally unstable. In chapter 13 of our book, we add to this evidence and show that the correlation between the return on foreign exchange and the forward premium is sometimes negative, sometimes positive, and sometimes insignificantly different from zero.

Acknowledging the importance of temporal instability goes a long way toward resolving the forward-rate “puzzle.” A returns process that gives rise to both negative and positive correlations with the forward premium implies that betting against the forward rate will be profitable during some time periods but not in others. We show that a trading rule based on betting against the forward rate does not deliver significant profits over the modern period of floating in the major currency markets.

Because the contemporary approach has led economists to construct fully predetermined, mostly invariant models of foreign exchange returns that ignore temporal instability, the “finding” of a negative correlation between returns and the forward premium has led them to conclude that there is easy money to be made in the foreign exchange market. But, since the correlation is sometimes negative and at other times positive, fully predetermined trading rules based on the forward rate do not deliver profits. As in the case of the disjunction between the exchange rate and macroeconomic fundamentals, the forward-rate “puzzle” is another artifact of the epistemological flaws inherent in the contemporary approach. The fact that there are literally hundreds of studies attempting to explain this “puzzle” provides an example par excellence of how contemporary economics’ insistence on sharp predictions has misdirected research and impeded its progress.

⁹⁶The forward premium depends on the difference between the forward and spot exchange rates.

⁹⁷See chapter 8 of our book for a discussion of this literature

16 Coming to Terms with Imperfect Knowledge

The research program of contemporary economics is predicated on the belief that it is possible to fully prespecify economic change over periods of time as long as decades. The premise that seems, at least implicitly, to motivate this mechanistic way of modeling market outcomes is that there exists a fully predetermined causal mechanism that underpins actual behavior on the individual and aggregate levels.

But, forecasting behavior on the part of purposeful individuals alters the causal mechanism that underpins market outcomes in ways—and at points in time—that cannot be fully prespecified. Moreover, changes in the social context, including the evolution of institutions, values, and norms, are all important in engendering temporal instability in causal relationships in real-world markets.

If change in capitalist economies is not governed by a fully predetermined causal mechanism, then attempting to explain individual behavior and aggregate outcomes with representations that presume the existence of such a mechanism is misguided. It is not surprising, then, that the contemporary approach has had great difficulties in discovering the “mechanics of economic development” (Lucas, 2002, p. 21) in many markets where profit-seeking inherently involves coping with ever-imperfect knowledge.

Beyond the contemporary research program’s empirical difficulties, its reliance on fully predetermined models precludes the possibility of building any coherent economic theory of aggregate outcomes based on plausible micro-foundations. Remarkably, what economic theory requires to escape extant approaches’ epistemological flaws is also required to overcome their empirical difficulties: economists must give up their insistence on sharp predictions.

Economics calls for a new approach that represents individual behavior and aggregate outcomes mathematically, and that, at the same time, refrains from fully prespecifying economic change. IKE offers a way to take up this task.

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