LEADERSHIP, TRUST, AND POWER: DYNAMIC MORAL HAZARD IN HIGH OFFICE

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Abstract: We consider a model of governors serving a sovereign prince, who wants to deter them from corruption and rebellion. Governors must be penalized when they cause observable crises, but a governor's expected benefits must never go below the rebellion payoff, which itself is better than what any candidate could pay for the office. Governors can trust the prince's promises only up to a given credit bound. In the optimal incentive plan, compensation is deferred until the governor's credit reaches this bound. Each crisis reduces credit by a fixed penalty. When a governor's credit is less than one penalty from the rebellion payoff, the governor must be called to court for a trial in which the probability of dismissal is less than 1. Other governors must monitor the trial because the prince would prefer to dismiss and resell the office. A high credit bound benefits the prince ex ante, but in the long run it generates entrenched governors with large claims on the state. Low credit bounds can cause the prince to apply soft budget constraints, forgiving losses and tolerating corruption for low-credit governors.

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Why is the Exchequer so called? ...Because the table resembles a checker board... Moreover, just as a battle between two sides takes place on a checker board, so here too a struggle takes place, and battle is joined chiefly between two persons, namely the Treasurer and the Sheriff who sits to render account, while the other officials sit by to watch and judge the proceedings.

Richard FitzNigel, Dialogue of the Exchequer (c. 1180).

1. Introduction.

To understand the structures of government, it can be helpful to consider the history of their development (Finer, 1997). Among the early roots of the Western political institutions that forged the modern world, the Court of the Exchequer stands out from the twelfth century as the central institution underlying the growth of royal power in medieval England (Warren, 1973, ch 6-8). Our understanding of government is incomplete if we do not recognize why such an institution should be so important in the foundations of the state.

The Exchequer maintained accounts of the king's transactions with his sheriffs, who were the primary agents of royal power in the counties of England. But such transactions were not merely recorded at the Exchequer. The rules and procedures of the Exchequer required that the sheriffs' transactions must be witnessed in detail by a panel of high officials and barons of the Exchequer. Thus the Court of the Exchequer was designed so that, if the king needed to punish one of his sheriffs, the grounds for such punishment could be certified by a broad group of high governmental officials.

Institutions with similar functions can be found in successful political systems throughout history. In ancient Rome, the Senate was an institutional forum where rights of senior government officials could be protected. As early as 1500 B.C.E., the Hittite king Telipinu created a similar institutional structure. Telipinu noted that the rise of the Hittites had depended on unity and trust among the royal family, so that Hittite kings could rule their empire by sending members of the royal family to govern the conquered provinces. But in later generations, royal kinsmen began to plunder their provinces or rebel against the king, with increasing frequency.

Telipinu proposed to solve these problems by a new constitutional rule: that the king should never punish any member of the royal family without a formal trial before the assembled royal council or *Panku* (van den Hout, 1997, Beckman, 1982). So this ancient Hittite council, like the medieval Exchequer, was an institution designed to help maintain trust between the king and those on whom he relied as the regional agents of state power.

Agency incentive problems are fundamental to the constitution of any political system. Constitutional rules are enforced by the actions of political leaders and government officials, who must be motivated by an expectation of rewards and privileges as long as they fulfill their constitutional responsibilities. So the survival of any political system depends on its providing appropriate incentives for political agents to take actions that may be subject to moral-hazard temptations and imperfect observability. High officials of government, such as governors and ministers, have particularly great power that can be abused for personal enrichment of the officials. To deter such abuse of power, the leader of the government must be able to assure high officials that loyal service will bring greater rewards. But such rewards are costly for the state, and the leader would prefer to minimize them.

It has been well understood, since Becker and Stigler (1974), that optimal solutions to such dynamic agency problems generally involve deferring or back-loading rewards as much as possible. When a promised reward is held back for later payment, it can be used as a motivation for loyal service over a longer period of time, by threatening to deny payment altogether if there is evidence of misbehavior. In the optimal incentive plan for the Becker-Stigler model, a new official is required to pay a large bond to the state when she is promoted to office, and this bond is to be repaid to the official when she retires if there is no evidence of malfeasance by her.

Similar dynamic agency models have been considered by Shapiro and Stiglitz (1984) and Akerlof and Katz (1989), but they assumed that job candidates are unable to post such ex-ante bonds for promotion into valuable offices. As in the Becker-Stigler model, they find that officials' good behavior must be motivated by promises of large future rewards in office. But in their models, the employer (here the state) must incur the expected cost of an unfunded liability whenever a candidate is promoted into such an office.

Ex post, however, the payment of deferred rewards for past service (or the repayment of bonds) may be seen as a costly burden on the state, and a leader might be tempted to repudiate such debts. Becker and Stigler explicitly assumed away any question of agents' bonds being falsely withheld and confiscated by the state, but of course such confiscation would materially benefit the head of state. An absolute monarch would have difficulty making a credible commitment to not confiscate such bonds without some constitutional constraint to bind him.

Thus, we find a central moral-hazard problem in government: the problem of guaranteeing to high government officials that the leader of the state will not deny his debts to them. To be effective, a political leader must be like a banker, whose promises are trusted and valued as rewards for current service. This central moral-hazard problem is fundamental to the nature of political leadership and to the constitution of the state.

In this paper, we consider a variant of the Becker-Stigler model that is designed to highlight the difficulties of this central moral-hazard problem in the leader's control of powerful administrative officials. (A different model, focusing on the leader's debts to the captains who put him in power, is considered by Myerson, 2008.) As in the Becker-Stigler model, we assume here that agents must always be deterred from corrupt abuse of power, where corruption provides hidden benefits to the agent but generates a positive probability of costly crises which the principal can observe. Four features may distinguish our variant from previous versions.

First, we drop the common assumption that an agent can guarantee herself a good record by good behavior. The accidental crises that are taken as evidence of corruption here can also occur with a positive (but smaller) probability even when an official is actually serving correctly. So under an incentive plan that successfully deters corrupt abuse of power, officials who have served correctly may nevertheless have bad outcomes, and so must face a positive probability of being dismissed without the promised rewards that motivated their good behavior. Thus, punishment of officials must occur with a positive expected frequency in our model.

Second, we adopt here an intermediate position between the different assumptions about job candidates' ability to pay for promotion to high office. We assume that job candidates can pay some positive amount for promotion to a valuable high office, but do not have sufficient funds to pay the full expected value of their promotion. This intermediate case creates a dynamic tension in the leader's incentives. When new officials cannot pay for the full value of the bonuses that they will expect in office, the leader's ex ante expected net cost is reduced by promising an employment policy that minimizes the expected turnover of his officials. But when new officials can pay some positive amount for their promotion to office, the leader actually profits ex post whenever he dismisses an official without rewards and promotes a new candidate. This tension between the ex ante optimal policy and ex post profit opportunities implies that an effective leader may need to commit himself by creating institutions that constrain and regulate his ability to opportunistically dismiss officials.

Third, in addition to the possibility of hidden corruption by officials, we admit a second way that they can abuse their power, by open rebellion, which could become optimal for them if they knew that they were about to be punished. The need to deter both hidden corruption and open rebellion here compels the leader to use a randomized strategy in judging his high officials. But it is difficult for others to verify that such a randomized judgment strategy has been implemented correctly, unless they can actively monitor the random process by which the judgment has been determined. Any judicial process that is constrained to allow such monitoring can be interpreted as a formal trial in an institutionalized court. So our derivation of regulated randomized dismissals can be understood as a theory of formal legal institutions.

Fourth, to introduce limits on the credibility of the leader's promises in the simplest possible way, without introducing multilateral game-theoretic complexity into our principal-agent problem, we add a parametric bound on the value of the deferred compensation that the leader can be trusted to owe any one official. We show that raising this bound always improves the leader's ex ante expected value. But a high trust bound implies that, in the long run, officials will become entrenched in office and will accumulate large expensive claims on the budget. Thus, we argue that the solution to central moral-hazard problems may impel a political leader to create formal institutional structures and procedures that regulate his judgment of high officials and effectively enroll them into a privileged aristocracy.

Finally, in section 8, we will introduce the possibility of tolerating corruption, which will require us to take account of the costs of crises for the leader. We find that, in some cases, low credit bounds can cause soft budget constraints to become optimal for the leader, so that a governor's losses may be forgiven and their corruption may be tolerated when credit is low.

2. A dynamic agency model of governors

To analyze the problems of motivating high government officials, we consider a dynamic agency model in which a high official, whom we may call a governor, serves under a political leader, whom we may call the prince. For clarity, we use female pronouns for governors and male pronouns for the prince. At any point in continuous time, the governor has three options: she may serve correctly as a good governor, or she may act corruptly, or she may openly rebel. The option of rebellion may also be interpreted as intensively looting the province and then fleeing abroad with the resulting treasures. We let D denote the expected total payoff to a governor when she rebels. The prince could observe any such rebellion.

The governor's choice among her other two alternatives, of good service or corruption, cannot be directly observed by the prince, but the prince can observe certain crises that may occur in the governor's province. When the governor serves correctly, crises will occur in her province as a Poisson process with rate α. On the other hand, when the governor acts corruptly, crises will occur in her province as a Poisson process with rate β, and the corrupt governor will also gain an additional secret income worth γ per unit time. That is, in any short time interval of length ε, if the governor is serving correctly then the probability of a crisis during this interval is $1-e^{-\alpha\varepsilon} \approx \alpha\varepsilon$; but if the governor is acting corruptly then the probability of a crisis during this interval is $1-e^{-\beta\varepsilon} \approx \beta\varepsilon$, and the corrupt governor would get an additional secret income worth $\gamma\varepsilon$ during this interval. We assume that $\beta > \alpha$.

We assume that the governor observes any crisis in her province shortly before the prince observes it. After any crisis, the governor can make a short visit to the prince's court, and the governor cannot rebel during such a short visit. The lengths of these short intervals may be considered as infinitesimals in our continuous-time model.

The possibility of profiting from rebellion or corruption can make the office of governor very valuable, but new candidates for promotion to governor have only some limited wealth which we denote K. We assume that the potential profit for a rebellious governor is greater than what any candidate could pay for the office; that is, $K < D$.

We assume that the prince's regime would suffer a large expected cost from any crisis or rebellion, and so the prince wants to always deter governors from rebellion or corruption. So we seek an incentive plan that make a governor always prefer to serve correctly.

The prince may derive some advantage from deferring payments to a governor, but the prince's temptation to sack a governor increases with the debt owed to her. To describe the bounds on trust of the prince, we let H denote the largest credit owed to a governor that the prince could be trusted to pay. If the prince's debt to a governor ever became larger than H, then the prince would abuse his own power, to eliminate the governor and the debt.

We assume that each individual is risk neutral and discounts future payments at the rate δ per unit time. Thus, our simple model is characterized by the seven parameters $(D,\alpha,\beta,\gamma,K,H,\delta)$, which are all assumed to be positive numbers. In section 8 we will introduce a parameter L to denote the prince's cost from any crisis, but for now we may simply assume that this cost is sufficiently large that the prince will always want to deter corruption, to keep the expected cost of crises always constant at its minimal value (αL per unit time).

3. Feasible incentive plans

We consider incentive plans in which the treatment of the governor at any time t depends on the past history only through a state variable u(t) that is the expected present discounted value of the future rewards that the prince owes to the current governor. This value u(t) may be called the governor's <u>credit</u> at time t. With the recursive structure of the prince's optimization problem, an optimal plan should, at any time t, minimize the expected present-discounted value of the prince's future costs subject to the constraint that the governor's expects to get u(t) by serving correctly and could not expect to get more by acting corruptly or by rebelling.

Here a credit of 0 denotes the level of expected payoffs that an individual could earn in competitive labor markets, outside the government. So a governor may expect to get credit 0 after leaving office, if she departs without severance pay or corporal punishment.

The state of a governor's credit can never be greater than H, the given bound on trust of the prince. Let G denote the lowest possible credit that a governor can have when serving in office. We must have $G \geq D$ because, to deter rebellion, a governor in office can never have a credit less than the value of rebellion D. We will see, however, that this lower bound G on a governor's credit must actually be strictly greater than D.

To describe how the prince's incentive plan treats a governor in office, let y(u) denote the governor's expected rate of pay per unit time, let $\theta(u)$ denote the rate of growth of the governor's credit per unit time, and let $\pi(u)$ denote the expected penalty that would be subtracted from the governor's credit if a crisis occurred, at a time when the governor's credit is u. That is, for any small $\varepsilon>0$, in any short time interval from t to $t+\varepsilon$ where the governor begins with credit $u(t)=u$, the governor's expected pay during this interval is approximately $\varepsilon y(u(t))$, the governor's expected credit at time $t+\varepsilon$ if no crisis occurs in the interval $[t,t+\varepsilon]$ is approximately

$$E(u(t+\varepsilon)|\ u(t)=u, \text{ no crisis occurs in } [t,t+\varepsilon]) \approx u+\varepsilon\theta(u),$$

to a first-order approximation in ε. Similarly, the governor's expected credit at time $t+\varepsilon$ if a crisis occurs in the interval $[t,t+\varepsilon]$ is approximately

$$E(u(t+\varepsilon)|\ u(t)=u, \text{ a crisis occurs in } [t,t+\varepsilon]) \approx u-\pi(u).$$

We assume that a new governor can be asked to pay her entire wealth K for the privilege of being promoted to a high office that is worth strictly more than K; but after taking this maximal ex-ante payment, the prince cannot demand any further personal payments from a governor. So for any feasible credit u, the pay rate $y(u)$ must be nonnegative:

(1) $$y(u) \geq 0, \ \forall u\in[G,H].$$

The governor's credit cannot grow larger than the upper bound H on trust of the prince, and so the credit growth rate $\theta(H)$ must be nonpositive at H:

(2) $$\theta(H) \leq 0.$$

To a first-order approximation in ε, the expected t-discounted value of payoffs for a governor with credit u must satisfy the recursive formula

$$u \approx \varepsilon y(u) + (1-\varepsilon\delta)\{(1-\varepsilon\alpha)[u + \varepsilon\theta(u)] + \varepsilon\alpha[u-\pi(u)]\}$$
$$\approx u + \varepsilon[y(u)-\delta u+\theta(u)-\alpha\pi(u)].$$

Thus, for the expected t-discounted value to actually equal the given credit u, a feasible incentive plan must satisfy the promise-keeping constraints:

(3) $$0 = y(u) + \theta(u) - \alpha\pi(u) - \delta u, \ \forall u\in[G,H].$$

On the other hand, if a governor with the credit u in [G,H] began acting corruptly for some short time interval ε then the governor could get an expected t-discounted value that has the first-order approximation:

$$\varepsilon y(u) + \varepsilon\gamma + (1-\varepsilon\delta)\{(1-\varepsilon\beta)[u + \varepsilon\theta(u)] + \varepsilon\beta[u-\pi(u)]\}$$
$$\approx u + \varepsilon[y(u)+\gamma-\delta u+\theta(u)-\beta\pi(u)],$$

because corruption would give her a secret additional income at the rate γ, but crises with penalty

π would occur at the probabilistic rate β instead of α. So to deter corruption, the incentive plan must satisfy the moral-hazard constraint

$$y(u) + \gamma + \theta(u) - \beta\pi(u) - \delta u \le y(u) + \theta(u) - \alpha\pi(u) - \delta u, \ \forall u \in [G,H].$$

Thus, given that $\beta > \alpha$, a feasible incentive plan that deters corruption at all times must satisfy

(4) $$\pi(u) \ge \gamma/(\beta - \alpha), \ \forall u \in [G,H].$$

Let τ denote this lower bound on the expected penalty $\pi(u)$ that is imposed on a governor after a crisis, that is, $\tau = \gamma/(\beta - \alpha)$.

Recall that the governor is among the first to observes any crisis and could immediately rebel then. So to deter rebellion after any crisis, the incentive plan must always satisfy

(5) $$u - \pi(u) \ge D, \ \forall u \in [G,H].$$

Together with (4), this implies that a governor cannot have credit less than $D+\gamma/(\beta-\alpha)$ while serving in office. In fact, we can show that this value is the lowest credit that a governor can expect while serving in office and be consistently deterred from corruption and rebellion. Obviously the trust bound H cannot be less than this value, if corruption and rebellion are to be successfully deterred. That is, our parameters must satisfy

$$H \ge D+\tau = D+\gamma/(\beta-\alpha).$$

When the governor's credit u is near the lower bound G, a crisis could cause her expected credit to drop to a value $\hat{u} = u - \pi(u)$ that is less than G. By (5) we have $\hat{u} \ge D$, and so she still does not want to rebel, but there is a danger that another such penalty after another crisis could indeed incite her to rebellion. Thus, a governor whose credit drops down between D and G after a crisis must be immediately called to the prince's court for a trial, which must have the effect of either dismissing the governor with some probability $q(\hat{u})$ or else reinstating the governor to some credit level $g(\hat{u}) \ge G$. Recall that the credit 0 denotes the expected level of payoffs that a former governor would get as a normal citizen after dismissal. But the outcome of the trial might also include some payment to the governor or some corporal punishment of the governor. Let $Y(\hat{u})$ denote the expected value of any payment that the governor might get in this trial when she enters it with credit $\hat{u}$. Then the promise-keeping constraint for a trial of a governor with post-crisis credit $\hat{u}$ is

(6) $$\hat{u} \le (1-q(\hat{u}))g(\hat{u}) + Y(\hat{u}), \ \forall \hat{u} \in [D,G).$$

If condition (6) were satisfied with a strict inequality, then the governor's expected payoff from the trial could be made to exactly match the promised credit $\hat{u}$ by a corporal punishment that hurts the governor as much as a cost of $(1-q(\hat{u}))g(\hat{u})+Y(\hat{u})-\hat{u}$, thus canceling out any surplus on the right-hand side of (6). We assume that such a corporal punishment could hurt the governor but would have neither benefit nor cost for the prince. For feasibility, we must also have

(7) $$Y(\hat{u}) \geq 0,\ \ G \leq g(\hat{u}) \leq H,\ \ 0 \leq q(\hat{u}) \leq 1,\ \ \forall \hat{u} \in [D,G).$$

As noted above, conditions (4) and (5) directly imply that the lower bound G on the credit for a governor in office cannot be less $D+\gamma/(\beta-\alpha)$. In fact there exist feasible incentive plans, satisfying (1)-(7), in which the range of possible payoffs for a governor in office actually includes this lower bound $D+\gamma/(\beta-\alpha)$. It suffices here to note that the plan described below in (14)-(18) is one such feasible plan. So we can henceforth take the lower bound on the credit for a governor in office to be

(8) $$G = D+\tau = D+\gamma/(\beta-\alpha).$$

4. The optimal incentive plan

For any possible credit level u, let V(u) denote the minimal expected present-discounted value of the prince's net cost of paying governors in this province, given that the current governor is owed a credit of u. Here V(0) denotes the expected present-discounted value of the prince's net cost when there is no governor in office (that is, when the previous governor has been dismissed to credit 0 and a new governor needs to be appointed). We now provide recursive optimality conditions to characterize this value function V(•) with the optimal incentive plan.

At any time, a governor with credit u could be offered a lottery that would change her credit to a random variable $\tilde{u}$ with expected value $E(\tilde{u}) = u$, which would be just as good for the governor because she is assumed to be risk neutral (her credit is measured in von Neuman Morgenstern utility units). But as the prince's policy is already optimal, $E(V(\tilde{u}))$ cannot be strictly less than V(u), and so V(u) must be a convex function of u. As a convex function, V is differentiable almost everywhere. If the slope $V'(u)$ were ever greater than 1 then the prince could reduce his expected cost by buying down the governor's credit by some payment y such that $y+V(u-y) < V(u)$. If the slope $V'(u)$ were ever negative, then the prince could reduce his expected cost simply by increasing the governor's credit (after subjecting the governor to some corporal punishment that cancels out her expected gain, if necessary). Thus, we have:

(9) V is a convex function, and its slope V' is never less than 0 or greater than 1.

The credit u=0 would correspond to a governor who has just been dismissed, but then the prince would then need to appoint a new governor to some initial credit $g \geq G$. The new governor's payment of K for promotion would reduce the prince's expected costs from V(g) to V(g)−K, and so

(10) $$V(0) = \text{minimum}_g\ V(g)-K \ \text{ subject to }\ G \leq g \leq H.$$

When a governor is owed any credit u in the interval [G,H), applying the incentive plan (y,θ,π) over a short time interval of length ε would give the prince an expected discounted cost that is, to a first-order approximation,

$$\varepsilon y(u) + (1-\varepsilon\delta)[(1-\varepsilon\alpha)V(u+\varepsilon\theta(u)) + \varepsilon\alpha V(u-\pi(u))]$$
$$\approx V(u) + \varepsilon[y(u) + \theta(u)V'(u) + \alpha V(u-\pi(u)) - \alpha V(u) - \delta V(u)].$$

When the bracketed term is minimized over all feasible policies satisfying conditions (1), (3), and (4), it must go to 0, because V(u) itself is the prince's minimal expected discounted cost when the credited owed is u. Recall from (4) that, to deter corruption, the penalty after a crisis must be at least $\tau=\gamma/(\beta-\alpha)$. So the value function for the optimal policy must satisfy the recursive optimality condition:

(11) $$\forall u\in[G,H),\ \delta V(u) = \text{minimum}_{y,\pi,\theta}\ y+\theta V'(u)+\alpha[V(u-\pi)-V(u)]$$
$$\text{subject to }\ y \geq 0,\ \pi \geq \tau,\ \theta = \delta u+\alpha\pi-y.$$

At u=H, we must add the constraint that $\theta\leq 0$ from (2), and this condition then becomes:

(12) $$\delta V(H) = \text{minimum}_{y,\pi,\theta}\ y+\theta V'(H)+\alpha[V(H-\pi)-V(H)]$$
$$\text{subject to }\ \theta \leq 0,\ \pi \geq \tau,\ y =\delta H+\alpha\pi-\theta \geq 0.$$

Finally, at any post-crisis credit level $\hat{u}$ that is less than G, an incentive plan with (q,g,Y) as in (6) and (7) would make the prince's expected cost

$$Y(\hat{u})+(1-q(\hat{u}))V(g(\hat{u}))+q(\hat{u})V(0).$$

Thus we get the following the recursive optimality condition for cases where a crisis has caused a governor's credit to drop below G:

(13) $$\forall\hat{u}\in[0,G),\ V(\hat{u}) = \text{minimum}_{Y,q,g}\ Y+(1-q)V(g)+qV(0)$$
$$\text{subject to }\ Y\geq 0,\ 0\leq q\leq 1,\ G\leq g\leq H,\ Y+(1-q)g \geq \hat{u}.$$

We can now characterize the optimal incentive plan that satisfies these recursive optimality conditions (9)-(13). In this optimal plan, a new governor always begins with the smallest permissible credit G, and so equation (10) becomes

(14) $g(0) = G = D+\gamma/(\beta-\alpha)$ and $V(0) = V(G)-K$.

After any crisis, the governor's expected credit is decreased by the smallest penalty $\tau = \gamma/(\beta-\alpha)$ that satisfies the incentive constraint (4) against corruption:

(15) $\pi(u) = \tau = \gamma/(\beta-\alpha), \ \forall u\in[G,H]$.

The governor's compensation is maximally deferred, so that pay y(u) becomes positive only when the governor's credit u reaches the upper bound H:

(16) $y(u) = 0$ and $\theta(u) = \delta u+\alpha\tau, \ \forall u\in[G,H)$,

(17) $\theta(H) = 0$ and $y(H) = \delta H+\alpha\tau$.

If the governor's expected credit drops to some $\hat{u}<G$ after any crisis then the governor is either reinstated to credit G, with probability $\hat{u}/G$, or is dismissed, with probability $q = 1-\hat{u}/G$:

(18) $Y(\hat{u}) = 0, \ g(\hat{u}) = G, \ q(\hat{u}) = 1-\hat{u}/G, \ \forall\hat{u}\in[D,G)$.

<u>Theorem 1.</u> The optimality conditions (9)-(13) for minimizing the expected discounted net cost of paying governors in the province are satisfied by the incentive plan described above in equations (14)-(18). So the optimized value function V satisfies the following equations

(19) $V(\hat{u}) = V(0) + \hat{u}K/G, \ \forall\hat{u}\in[0,G]$.

(20) $V'(u) = [(\delta+\alpha)V(u) - \alpha V(u-\tau)]/(\delta u+\alpha\tau), \ \forall u\in[G,H]$

(21) $(\delta+\alpha)V(H) - \alpha V(H-\tau) = \delta H + \alpha\tau$, and so $V'(H) = 1$.

The proof of Theorem 1 is deferred to the Appendix. But the computational tractability of this model can be indicated by following Lemma, also proven in the Appendix.

<u>Lemma 1.</u> An increasing convex function Ψ can be uniquely characterized by:

(22) if $u \le G$ then $\Psi(u) = 1$,

(23) if $u \ge G$ then $\Psi'(u) = [(\delta+\alpha)\Psi(u) - \alpha\Psi(u-\tau)]/(\delta u+\alpha\tau)$.

This function $\Psi(u)$ is continuous, but its derivative $\Psi'(u)$ has one discontinuity at u=G, where equation (23) yields the right derivative (the slope of $\Psi(u)$ at u=G for small increases to $u\ge G$). When $u\ge G$, the derivative $\Psi'(u)$ is strictly increasing in u. Then prince's optimal value function V can be computed from this function Ψ by the equations

$$V(0) = (1-K/G)/\Psi'(H) = (1-K/G)(\delta H+\alpha\tau)/[(\delta+\alpha)\Psi(H)-\alpha\Psi(H-\tau)],$$

$$V(u) = uK/G + \Psi(u)V(0) = uK/G + \Psi(u)(1-K/G)/\Psi'(H), \ \forall u\in[0,H].$$

Equations (20)-(21) in Theorem 1 imply that $V'(H) = 1$, which is used in the derivation of Lemma 1. In Lemma 1, the formula for V(0) and the fact that $\Psi'(u)$ is strictly increasing in u when $u \geq G$ also imply the following important comparative-statics result.

Theorem 2. When $H \geq G$ and all other parameters are held fixed, an increase of the trust bound H would strictly decrease V(0), the expected present-discounted value of the prince's cost when a new governor is first appointed.

5. An example

To illustrate this solution, let us consider a numerical example where the discount rate is $\delta = 0.05$, the normal crisis rate is $\alpha = 0.1$, the corrupt crisis rate is $\beta = 0.3$, the corrupt income rate is $\gamma = 1$, the expected value of rebellion is $D = 5$, candidates have payable assets worth $K = 1$, and the upper bound on the prince's debt to any governor is $H = 25$. With these parameters, the crisis penalty is $\tau = \gamma/(\beta-\alpha) = 5$, and the minimal credit for governors is $G = D+\tau = 10$.

Figure 1 shows the how the prince's expected cost V(u) depends on the debt u that is owed to the current governor. The prince's expected cost when appointing a new governor would be $V(0) = V(G)-K = 10.44$ for this example. The slope of the cost function V' is 0.1 from u=0 to u=G=10, but there is a kink at u=10 where the slope jumps to 0.622, and then the slope increases continuously to 1 at H=25. When V is extended to negative credits, there is another kink at 0, because $V(u) = V(0) = 10.44$ for all $u<0$.

[Insert Figures 1 and 2 about here]

Figure 2 shows how the prince's expected cost $V(G)-K$ would change if the upper bound H were changed, holding fixed all other parameters of this example. The curve in Figure 2 is computed by applying the formula $V(G)-K = (1-K/G)/\Psi'(H)$, where $\Psi(u)$ can be extended to all $u \geq G$ by the differential equation (23). The curve cannot be extended to $H<G$, because the incentive constraints (4) and (5) would become impossible to satisfy if the credit bound H were less than the minimal credit level G from (8). Increasing the credit bound H would decrease the prince's expected cost $V(G)-K$ when a new governor is appointed, here from an expected cost of 18 when the credit bound is 10, to an expected cost of 10.42 when the credit bound approaches

infinity. This negative slope in Figure 2 is implied by Theorem 2 above.

6. Properties of the long-run steady state

Let us consider now the long-run steady-state distribution of the governor's credit u under the optimal incentive plan. Let F denote the strict cumulative distribution of u in this stationary distribution. That is, F(u) denotes the probability that the governor's credit is strictly less than u. The governor almost always has a credit in the interval [G,H]. (The governor's expected credit may be between D and G during post-crisis trials, but they are assumed to be infinitesimally brief.) So this strict cumulative F satisfies

$$F(u) = 0 \text{ if } u \leq G,$$

$$F(u) = 1 \text{ if } u > H.$$

We can have $F(H) < 1$, and $1-F(H)$ denotes the probability that the governor's credit is H.

Now consider any u between G and H. $F(u+\tau)-F(u)$ is the probability of a governor being at u or above by less than the penalty amount τ, and the probability of incurring a penalty τ during a short interval of length ε is approximately $\varepsilon\alpha$. So to a first-order approximation in ε, the probability of the governor's credit decreasing across u during a short interval of length ε is $\varepsilon\alpha[F(u+\tau)-F(u)]$. $F'(u)$ is the probability density at u, and near u the governor's credit increases between crises at the rate $u'(t) = \delta u+\alpha\tau$. So the probability of a governor being below u by a small amount $\varepsilon(\delta u+\alpha\tau)$ such that the governor's credit would increase across u during a short interval of length ε, is $\varepsilon(\delta u+\alpha\tau)F'(u)$, to a first-order approximation in ε. In the stationary steady-state distribution, the probability of crossing u in each direction must be the same. Thus, F must satisfy the differential equation

$$(24) \qquad (\delta u+\alpha\tau)F'(u) = \alpha[F(u+\tau)-F(u)], \ \forall u\in[G,H].$$

To compute F, we may let the function Ω be defined by

$$\Omega(u) = 0 \ \forall u>H,$$

$$\Omega(H) = 1,$$

$$-\Omega'(u) = \alpha[\Omega(u)-\Omega(u+\tau)]/(\delta u+\alpha\tau) \ \forall u\leq H.$$

Notice that Ω satisfies the same differential equation as F but with different boundary conditions. This differential equation can used to numerically compute $\Omega(u)$ continuously for all $u\leq H$. But $[1-F(u)]/[1-F(H)]$ satisfies the same conditions as $\Omega(u)$ when $u\geq G$, and so

$$\Omega(u) = [1-F(u)]/[1-F(H)], \ \forall u \geq G.$$

Then with F(G)=0, we get $\Omega(G) = 1/[1-F(H)]$, and so F can be computed from Ω by

(25) $$F(u) = 1-\Omega(u)/\Omega(G), \ \forall u \geq G.$$

The stationary distribution F has a continuous probability density

$$F'(u) = -\Omega'(u)/\Omega(G)$$

on the interval [G,H), but there is also has a discrete probability mass $1-F(H)$ at the upper bound H, where the governor gets paid.

Figure 3 shows the cumulative distribution F for our example from section 5, with α=0.1, β=0.3, δ=0.05, D=5, K=1, and H=25. In the stationary distribution, the probability of a governor being at the credit bound H is $1-F(H) = 0.68$.

[Insert Figure 3 about here]

The pay rate for governors at H is $\delta H+\alpha\tau = 0.05\times25 + 0.1\times5 = 1.75$ here. So in the stationary distribution, the prince's expected wage-expense rate is

$$(\delta H+\alpha\tau)(1-F(H)) = 1.19.$$

A crisis can cause a governor to be dismissed only when the governor's pre-crisis credit was between G and G+τ, which is between 10 and 15 for this example, but the stationary probability of this interval is F(15) = 0.015. The expected rate of dismissals in the stationary distribution is

$$\int_{u\in[G,G+\tau]} \alpha[1-(u-\tau)/G)] \, dF(u) = 0.00030,$$

So in the stationary distribution, the prince's expected discounted value of net wage costs (wage expenses, minus payments from new governors) is

$$(1.19-0.00030K)/\delta = 23.8.$$

This stationary discounted cost is much greater than the prince's expected discounted cost when he appoints a new governor, which is $V(G)-K = 10.44$ for this example. When a new governor is appointed, the prince can reduce the expected discounted value of his costs by planning to defer compensation until his debts reach the credit bound H. But then the prince's costs will become greater after the debt of H has been incurred, and the stationary distribution takes this ex-post perspective.

For comparison, if the credit bound H were at the smallest feasible level H = G = 10, holding fixed the other parameters of this example, then the prince would always pay his governor $y = \delta G+\alpha\tau = 1$, and governors would be dismissed at the expected rate $\alpha[1-(G-\tau)/G]$

= 0.05. So the prince's expected costs when would normally be worth $V(G) = (y-0.05K)/\delta = 19$, which would decline to $V(G)-K = 18$ when a new governor is appointed. Thus, a lower credit bound H would make the prince worse off ex ante, when a new governor is first appointed, but could make the prince better off ex post, in the long-run stationary distribution.

In this example, the prince's ex-post stationary expected cost rate actually declines as the credit bound H increases from 10 to 15, but it becomes increasing in H when H > 15. This result for large H can be shown to hold in broad generality. The following theorem tells us that, if H is large, then a governor's credit is unlikely to be far from H in the stationary distribution. (The proof can be found in the Appendix.)

<u>Theorem 3.</u> The stationary strict cumulative distribution F satisfies, for any integer $m \geq 0$,

$$\text{if } u \leq H-m\tau \text{ then } F(u) \leq [\alpha\tau/(\delta G+\alpha\tau)]^{m+1}.$$

When credit **u** is drawn from the stationary distribution F, its expected value satisfies

$$E(\mathbf{u}) \geq H-\alpha\tau^2/(\delta G),$$

and its probability of being at the upper bound H satisfies

$$P(\mathbf{u}=H) = 1 - F(H) \geq (\delta H - \alpha\tau^2/G)/(\delta H+\alpha\tau).$$

The bounds in Theorem 3 imply that the long-run stationary probability of being at the upper credit bound H goes to 1 as H becomes large, that is, $1-F(H)\to 1$ as $H\to\infty$. So when H is large, the leader is in the long run usually paying high wages $\delta H+\alpha\tau$ to each governor. The long-run expected pay rate $(\delta H+\alpha\tau)(1-F(H))$ is bounded below by $\delta H-\alpha\tau^2/G$, which goes to infinity as H becomes large.

We have seen (as in Figure 2) that the prince's expected discounted costs when a new governor is appointed are strictly decreasing in the upper bound H. Thus, even when the leader is secure in power and is as patient as his agents (discounting the future at the same rate δ), agency costs give the leader an incentive to accumulate large expensive debts to governors.

The governors' turnover rate is bounded above by the inequality

$$\int_{u\in[G,G+\tau]} \alpha[1-(u-\tau)/G]\, dF(u) \leq \alpha F(G+\tau).$$

The bounds in Theorem 3 imply that $F(G+\tau)$ goes to 0 as H becomes large. So when the prince has a high credit bound H, successful governors tend to become entrenched in office as a closed aristocracy. The prince prefers such a system that minimizes new entry into high office, because he expects to lose $G-K$ whenever he promotes an outside candidate who can only pay K for an position that is worth G.

7. Analysis of the solution

Whenever a new governor is appointed, the prince incurs a liability worth G in exchange for a smaller payment K, and so the prince's expected net cost from any promotion must be $G-K$. The prince's total expected discounted cost is equal to the expected present discounted value of his net costs from all future appointments. Thus, the optimal incentive plan for the prince should minimize the expected frequency of replacing governors.

In a model that is similar to ours, Akerlof and Katz found no advantage to delayed wage increases, but they assumed that crises do not occur when an official serves correctly. That is, in our terms, their model assumed that α would equal zero. But in our model, with $\alpha>0$, deferring a governor's compensation until the credit bound H is reached can help to reduce the prince's expected discounted cost by reducing the expected turnover of governors, because some crises can be punished by temporary suspension of wages instead of by dismissal.

Ex post, however, the (unanticipated) replacement of a governor who has been promised $u \geq G$ would yield benefits to the prince worth $K+V(u)-V(G) > 0$. Thus, although the prince wants ex ante to promise a low rate of turnover among his governors, they must always be suspicious that ex post he may try to find excuses to violate this promise and deny their promised rewards. In our model, the parameter H represents the bound on how much the prince can credibly promise any governor.

We have seen that, to deter his governors from corruption and rebellion, the prince's credit bound H cannot be less than the minimal credit level G that a loyal governor requires. If governors could not trust the prince to pay them future rewards worth D, then they would rebel immediately. If the prince's credit bound were between D and G, then he could deter governors from rebellion, but he could not penalize crises sufficiently to deter governors from corruption.

In the optimal incentive plan, by (18), the prince randomizes between dismissal and reinstatement of the current governor after crises when the governor's prior credit is below $G+\tau$. The threat of dismissal must be moderated by randomization here, because otherwise it would incite governors to rebel after crises. Within our model, any procedure in the prince's court that implements this randomization could be considered a "fair trial" for the governor. But in such a trial, the prince would actually prefer to dismiss rather than reinstate because, after a dismissal, he can get a new candidate to pay $K>0$ for promotion into the office of governor. Thus, the positive probability of reinstatement in such trials can be credible only if it is guaranteed by some institutional constraint on the prince.

The fairness of these trials must be actively monitored, because the correct outcome of the trial cannot depend only on the facts of the case but must also depend on some unpredictable element in the trial itself. If the correct outcome of a governor's trial depended predictably on the nature of the crisis in her province, then the governor would rebel before any trial that was to result in her dismissal, because the governor observes the crisis before she is called to court. Thus, the outcome of the trial must depend on unpredictable random events in the trial, and the correctness of the outcome can be verified only by people who have observed the process of the trial in detail.

Who can punish a sovereign prince for wrongly dismissing a governor in an unfair trial? We have not formally modeled the prince's payoffs here, but we have assumed that corruption and rebellion would be very costly for the prince, perhaps because they could terminate his reign. So governors and other high officials, as a group, have the power to punish such misbehavior by the prince. If the prince were known to have wrongly dismissed a governor in an unfair trial, then other governors could lose trust in future trials, so that they would rebel after any crisis. That is, the prince could be deterred from cheating a governor by the threat that his perceived credit bound would drop below G if such cheating were observed. Thus, within our model, the prince can guarantee the fairness of a governor's trials by inviting other governors to observe it. The effectiveness of the threat here depends on a sense of identity among the high officials who serve the prince, so that they would all lose faith in the prince's promises if he cheats any one of them. (See also Myerson, 2008, for further discussion of this crucial point.)

Other alternatives to randomized trials would be more costly to the prince in our model. Our analysis allowed for the possibility that governor's could be punished or given severance pay, but neither is used in the optimal solution. For example, consider the situation when a governor's expected credit drops to $u-\tau<G$ after a crisis. Dismissing the governor with severance pay $u-\tau$ would make the prince's expected cost $V(G)+u-\tau-K$, because a new governor will pay K for appointment to credit G. On the other hand, reinstating the old governor at the minimal credit G

after inflicting a corporal punishment that hurts her as much as an income loss of $G-(u-\tau)$ would make the prince's expected cost $V(G)$, given that such corporal punishments have no benefit to the prince. With $0<K<G$, both of these expected costs are strictly greater than the prince's expected cost $V(u-\tau) = V(G)+(u-\tau-G)K/G$ under the optimal plan with randomized dismissal. Severance pay is not optimal because it increases the expected rate of turnover, which is costly for the prince when $K<G$. Corporal punishment could decrease the expected rate of turnover but is not optimal because, when $K>0$, the prince would have to pay more to compensate governors for their anticipated risk of corporal punishment than the prince could gain by reducing turnover.

As an extreme case where trust in the prince is minimal, we may consider the case where the prince cannot be trusted to judge his governors unless his costs are completely independent of these judgments. Then to guarantee his indifference to questions of punishing or replacing governors, the prince could not charge any fee for promoting a new governor, and the prince's debt to a governor would have to be history-independent. In our model, these conditions correspond to the parametric case of $K=0$ and $H=G$. In such a case, the optimal incentive plan would reduce to paying the constant efficiency wage $y = \delta G+\alpha\tau$, where $\tau = \gamma/(\beta-\alpha)$ and $G = D+\tau$, and replacing the governor with probability $1-q = \tau/G$ whenever a crisis is observed in her province. The prince's expected cost is then

$$V(G) = (\delta G+\alpha\tau-K\alpha\tau/G)/\delta,\ \ V(0) = V(G)-K.$$

But in this case with $K=0$, unproductive corporal punishment could also be included in an optimal incentive plan. (Indeed, as the Hittite king Telipinu considered the danger of false accusations by the palace staff, he stipulated that those who were convicted by the council could be executed, but all their possessions should pass to their heirs without any expropriation.)

8. Tolerating low effort with a soft budget constraint

Now let us relax the assumption that the prince will never tolerate corruption. To endogenize the decision about whether deterrence of corruption is worthwhile, we need to take the cost of corruption into account. To keep things simple, we may assume that the prince's loss from corruption is entirely due to the increased rate of crises that it causes, where each crisis costs the prince some amount L. We continue to assume here that the prince's (unspecified) cost of rebellions is large enough that he wants to always deter rebellion, so the prince's trust bound cannot be less than the governor's rebellion payoff, that is, $H \geq D$. Also for simplicity, let us assume now that candidates for governor cannot pay anything for promotion to governor, that is, $K=0$.

What we have been calling corruption may be reinterpreted as low effort against crises, and good service is high effort. It will be convenient for us now to reinterpret γ as the governor's unmonitorable cost for high effort that can reduce the expected rate of crises (instead of as the governor's hidden income from corruption that would increase the rate of crises). So now the prince must pay an additional rate γ to the governor to cover these costs whenever high effort is demanded. To make sure that high effort would be economically efficient if it were observable, let us make the parametric assumption

$$\beta L > \alpha L+\gamma, \text{ that is, } L > \gamma/(\beta-\alpha) = \tau.$$

This γ expenditure allowance is not counted as income to the governor because she is supposed to be spending it on the high effort against crises, but there is a moral hazard problem because the prince cannot directly observe the governor's effort expenditures. So although the prince may demand high effort and pay the allowance necessary for it, the governor can always choose low effort and spend the γ allowance on her own consumption instead. The governor would prefer this low-effort alternative if her expected penalty for new crises were less than $\tau = \gamma/(\beta-\alpha)$.

Let W(u) denote the optimal expected present-discounted value of prince's costs when the governor has been promised credit u, including now the cost of crises and the effort cost γ which the prince must pay when the governor is supposed to choose high effort. So this value W(u) differs from the value V(u) that we analyzed above in that W(u) also includes the expected discounted cost of crises and high effort, which would be $(\alpha L+\gamma)/\delta$ in the case where the governor always maintains high effort.

As before, it is optimal for the prince to defer the payment of any income to the governor until her deferred credit reaches the trust bound H. As before, it is optimal for the prince, after any crisis, to subtract from the governor's credit the minimal penalty that can motivate the action that is currently demanded of the governor. When high effort is demanded, this minimal motivating penalty is $\tau = \gamma/(\beta-\alpha)$. But when low effort is demanded, the minimal motivating penalty is 0. So to guarantee that the governor's expected credit after a penalty will never go below her rebellion value D, high effort can be demanded only when the governor's credit u satisfies $u \geq G = D+\tau$. But now low effort could be demanded at any credit u satisfying $u \geq D$.

For any short time interval ε, when the governor with credit u between G and H is motivated to choose high effort, the prince's expected discounted cost is, to first-order approximation in ε,

$$W(u) \approx \varepsilon\alpha L + \varepsilon\gamma + (1-\varepsilon\delta)[(1-\varepsilon\alpha)W(u+\varepsilon(\delta u+\alpha\tau)) + \varepsilon\alpha W(u-\tau)]$$
$$\approx W(u) + \varepsilon[\alpha L+\gamma+(\delta u+\alpha\tau)W'(u) + \alpha W(u-\tau) - \alpha W(u) - \delta W(u)].$$

Here, as before, $\delta u+\alpha\tau$ is the credit growth rate between crises that must be given to satisfy expected promise-keeping to the governor. On the other hand, when low effort is motivated at any $u\geq D$, this expected discounted cost is

$$W(u) \approx \varepsilon\beta L +(1-\varepsilon\delta)W(u+\varepsilon\delta u) \approx W(u) + \varepsilon[W'(u)\delta u+\beta L-\delta W(u)].$$

Here δu is the credit growth rate that satisfies promise-keeping, as no crisis penalties are applied when low effort is expected. The optimal effort that the prince should demand is the one which yields the smaller expected discounted cost, and the bracketed coefficients of ε in this minimized cost must be equal to zero. So at any $u\geq G$, we have the recursive optimality condition

$$0 = \text{minimum}\ \{\alpha L+\gamma+(\delta u+\alpha\tau)W'(u)+\alpha W(u-\tau)-\alpha W(u)-\delta W(u),\ W'(u)\delta u+\beta L-\delta W(u)\}. \tag{26}$$

Notice that (26) is implicitly a differential equation for $W'(u)$, because $W'(u)$ must make the minimal expression in (26) equal to zero. The expression corresponding to high effort in (26) becomes zero when $W'(u)$ is equal to $\Omega_1(u)$ where

$$\Omega_1(u) = (\delta W(u) + \alpha W(u) -\alpha W(u-\tau) - \alpha L - \gamma)/(\delta u+\alpha\tau),\ \ \forall u\geq G.$$

The expression corresponding to low effort in (26) becomes zero when $W'(u)$ is equal to $\Omega_0(u)$, where

$$\Omega_0(u) = (\delta W(u) - \beta L)/(\delta u),\ \ \forall u\geq D.$$

Both expressions in (26) have positive coefficients for $W'(u)$, and so the optimal effort is the one which corresponds to the higher value of $W'(u)$. So the recursive optimality condition (26) is equivalent to

$$W'(u) = \text{maximum}\ \{\Omega_1(u), \Omega_0(u)\},\ \ \forall u. \tag{27}$$

The above definitions of $\Omega_1(u)$ and $\Omega_0(u)$ are applied only for credit levels u where the corresponding effort level can be motivated without any possibility of inciting a rebellion, that is, $u\geq G=D+\tau$ for the high-effort formula $\Omega_1(u)$, and $u\geq D$ for the low-effort formula $\Omega_0(u)$.

To extend these definitions to lower credit levels, notice that high effort can be demanded of a governor with credit less than G only if he is immediately promoted to credit G, and such a jump promotion can be optimal only over an interval where W is constant and W′ is zero. So let

$$\Omega_1(\hat{u}) = 0, \ \forall \hat{u}<G.$$

Similarly, low effort requires an instant promotion from any credit less than D, and so we let

$$\Omega_0(u) = 0, \ \forall \hat{u}<D.$$

With these extensions, the differential equation (27) can be applied at all u<H.

At the upper boundary where u=H, credit promotions are replaced by payments, and so the recursion expressions for high and low effort at u=H differ from (26) only in that W′(u) (as the prince's cost coefficient for the rate of increasing the governor's credit) must be replaced by 1 (as the prince's cost coefficient for the rate of pay to the governor). This substitution yields the boundary condition

(28) $$\text{maximum } \{\Omega_1(H), \Omega_0(H)\} = 1.$$

These optimality conditions (27) and (28) can be solved numerically by first guessing the initial expected cost W(0), integrating the differential equation (27) forward from u=0 to u=H, and then identifying the correct initial value W(0) by the boundary condition W′(H) = 1 from (28).

We can now characterize the prince's optimal incentive plan for his governors in this model. The optimal solution depends on whether W(0) is greater or less than $\beta L/\delta$.

When $W(0) \leq \beta L/\delta$, high effort is always demanded of the governor, and the optimal solution from Theorem 1 is applied (except that now the prince pays additional compensation at rate γ to cover the cost of high effort). So in this case, a new governor is started at credit $G=D+\tau$, suffers the penalty τ after any crisis, and has a positive probability of being dismissed whenever her credit drops below G. As observable losses can cause the governor to lose her position, this case may be called the <u>hard budget constraint</u> or <u>HBC case</u>.

When $W(0) > \beta L/\delta$, a new governor is started at credit D, and low effort is tolerated with no crisis penalties as long as the governor's credit is less than $G=D+\tau$. In this case, the governor is never dismissed, no matter how many costly crises she causes, and so this case may be called the <u>SBC case</u> or the <u>soft budget constraint</u> in the sense of Kornai (see Kornai, Maskin and Roland, 2003). In this case, while credit is less than both G and H, the governor's credit grows at rate $u'(t) = \delta u$, regardless of any crises. When the governor's credit becomes G or greater, then high effort is demanded and the crisis penalty τ is applied in this SBC case, just as in Theorem 1, and credit grows between crises at the rate $u'(t) = \delta u(t)+\alpha\tau$ as long as $u(t)<H$. When credit reaches the upper bound H, the governor is paid an income (above the cost of the demanded effort γ) at the rate $\delta H+\alpha\tau$ if $H \geq G$, or at the rate δH if $H < G$. So in this SBC case, the motivation for high effort (with credit G or more) is derived entirely from the promise of pay at credit H, as there is no threat of dismissal at the low end.

Thus, a costly crisis can cause a governor to be dismissed in the HBC case, but effort is always high. The governor is never dismissed in the SBC case, but effort is sometimes low. These results are summarized by the following theorem, proven in the Appendix.

<u>Theorem 4.</u> When low effort can be tolerated, the optimal incentive plan is characterized by one of two cases. When $W(0) \leq \beta L/\delta$, the HBC case applies: a new governor is started at credit $G=D+\tau$, and high effort is always demanded according to the optimal plan in Theorem 1, and the expected cost function W can be computed from V in Theorem 1 by the formula:

$$W(u) = V(u)+(\alpha L+\gamma)/\delta, \ \forall u \geq 0.$$

When $W(0) > \beta L/\delta$, the SBC case applies: a new governor is started at credit D, and the optimal policy tolerates low effort with no crisis penalties while the governor's credit u satisfies $D \leq u < G$; but the optimal policy demands high effort when $u \geq G$, and so the expected cost function satisfies $W'(u) = \Omega_0(u)$ when $D \leq u < G$, but $W'(u) = \Omega_1(u)$ when $G \leq u \leq H$. In the SBC case, costs are

$$W(u) = u + (V(u)-u)(\beta L-\alpha L-\gamma)/(\delta V(0)) + (\alpha L+\gamma)/\delta, \ \forall u \geq D,$$

and $W(0)=W(D)$. The SBC case applies when $(\beta L-\alpha L-\gamma)/\delta < V(0)$.

In the parametric case where H=G, we would get $V(0) = (\delta G+\alpha\tau)/\delta$, because motivating high effort while maintaining credit G would require a constant wage $\delta G+\alpha\tau$. With higher H, V(0) becomes smaller. Thus, if the prince's cost of crises satisfies $L < (\delta G+\alpha\tau+\gamma)/(\beta-\alpha)$ then the SBC case applies at H=G, but increasing the trust bound H may cause a switch to the HBC case. If $L \geq (\delta G+\alpha\tau+\gamma)/(\beta-\alpha)$ then the HBC case applies with any trust bound $H \geq G$.

To illustrate, let us consider examples where, as before, $\alpha=0.1$, $\beta=0.3$, $\gamma=1$, $\delta=0.05$, and $D=5$, but now $K=0$. To have $\alpha L+\gamma < \beta L$, the crisis cost L must satisfy $L > \tau = 5$.

For the lowest possible trust bound H=D, it is easy to verify that the prince's total discounted cost must be $W(0) = W(D) = D+\beta L/\delta = 5+6L$, which yields $\Omega_0(D) = (W(0)-\beta L/\delta)/D = 1$. In fact this solution applies with any H less than G.

Now consider the case of H=G=10, the smallest credit bound where high effort is possible. For the HBC case to hold with H=G, we need $\beta L/\delta \geq W(0) = (\delta G+\alpha\tau+\alpha L+\gamma)/\delta$, which is satisfied here if $L \geq (\delta G+\alpha\tau+\gamma)/(\beta-\alpha) = 10$. So with $L \geq 10$ and H=G here, the new governor would be started right away with credit G, and would always be expected to exert high effort, but would be dismissed with probability $\tau/G = 0.5$ whenever a crisis occurs. For smaller L between 5 and 10, the SBC case holds, and so the governor would start at credit D, and she would be expected to exert low effort until her credit grew to u=G after a growth period of length $LN(G/D)/\delta = 13.86$, but then she would be paid at rate $\delta G+\alpha\tau+\gamma=2$ for high effort until the next crisis (after which her pay and high efforts would again be suspended for a period of length 13.86). With such a SBC incentive plan, the governor would never be dismissed.

Now let the crisis cost be L=9, keeping other parameters as above. Then a low trust bound H near G=10 would put us in the SBC case, where $W(0)>\beta L/\delta$, and so the governor would never be dismissed. But increasing the parameter H would reduce the prince's expected cost W(0). When H>12.4 with L=9, the expected cost W(0) falls below $\beta L/\delta$, and so the hard budget constraint applies here. That is, when the trust bound H is greater than 12.4, governors are started at the initial credit level G, are always expected to exert high effort, and have a positive probability of dismissal if their credit ever drops below G after a crisis. Thus we see that increasing the upper bound on what the prince can credibly promise to the governors can switch the optimal incentive policy from one characterized by a soft budget constraint to one characterized by a hard budget constraint.

[Insert Figure 4 about here]

Figure 4 shows where the soft and hard budget constraints are optimal for different (L,H) pairs, keeping the other parameters kept as above. The boundary between the two regimes can be found, for any crisis cost L, by integrating the differential equation (27) forward from the initial condition $W(0)=\beta L/\delta$, and then finding the trust bound H that satisfies $W'(H)=1$ for equation (28) with this initial condition. For any L between 7.9 and 10, an increase of the trust bound H above G=10 can cause the optimal policy to switch from soft to hard budget constraint.

When the soft budget constraint is applied here, it is not caused by any intrinsic cost for the prince to dismiss a governor. Instead, we have found that a soft budget constraint can be caused by a low trust bound H. That is, the soft budget constraint can be caused by the prince's temptation to dismiss governors who claim large credits, to profit by canceling their claims.

9. Other assumptions about the prince's circle of trust

The analysis here has considered the prince's problem of filling a single governor's office, where the characteristics of this office (the moral-hazard opportunities associated with it) are given exogenously. Such governorships are assumed here to be uniquely powerful positions in society, because the wealth that a governor can earn ($G = D+\gamma/(\beta-\alpha)$) is substantially larger than what anyone could possibly earn outside of these offices (K). Our model may well be appropriate for a state that is naturally divided into a few large provinces, each of which requires an official to locally represent the authority of the state. But the validity of our assumptions may become weaker in a world with many different offices.

For example, suppose instead that there were many governorships, so that vacancies in such offices occur frequently. Under this assumption, when a governor deserves some $u-\tau < G$ after a crisis, the governor could be temporarily retired or furloughed, and the required expectation $u-\tau$ could be derived from her prospects of returning to high office when some future vacancy occurs. For such a plan to be feasible, the expected discount factor after the furlough period $E(e^{-\delta T})$ must match the probability of reinstatement $(u-\tau)/G$ in the randomized fair trials of our plan from section 4. Under this furlough plan, however, the prince must be constrained to respect the right of a former governor to return to high office, after an appropriate period in disgrace. Ex post, of course, the prince would prefer to disavow his implicit debt to the disgraced former governor and instead sell the office to a new candidate for K.

The prince could do even better when there are many different offices with greater and lesser opportunities for moral hazard. For example, suppose that a lower office exists where the lesser opportunities for corruption can be deterred by promising future rewards that are worth K when the official serves correctly. The prince would lose nothing by selling this office to an outside candidate for the amount K. But then, instead of paying the lower official, suppose that the prince simply increases her credit over time until the official's credit reaches amount G that a new governor needs. Then the prince could promote this official to governor without loss. In this state, since each promoted candidate pays the full value of her office, the prince would have no cost of turnover, and so the prince would derive no advantages from increasing the credit bound H above G and decreasing the rate of turnover below α. For this plan to work, however, the lower officials must share the higher officials' confidence that the prince will honor debts of size G or more to them. In effect, the lower officials must be members of the same circle of trust with the high officials.

Of course, if everybody could trust the prince to faithfully repay debts of size G or greater, then the prince could simply offer a savings plan where citizens would be invited to deposit their private wealth K along with all subsequent interest on these deposits, until they accumulate enough to pay the full value G for a governorship. Such savings accounts would invalidate our initial assumption of a gap between the greatest value K that common citizens can accumulate and the least value G that a governor must expect. But in this alternative world, the prince would be tempted to expropriate the citizens' deposits, just as he would prefer to disavow his implicit debt to an individual who has served without pay in a lower office.

Thus, the realism of our model depends on recognizing that, in many societies, individuals may have very different abilities to defend valuable claims against the state. Throughout history, many rulers have been able to hold political power without trust or support from the great mass of common people, but no ruler can hold power for long without the trust and support of the governors and captains who are the principal instruments of his power. That is, any successful political leader must be able to credibly promise large future rewards (at least G here) to the high officials of his government, but the same leader may be unable to credibly restrain his government from cheating common citizens of much smaller amounts. Indeed, the leader's need to maintain faith with his high officials may prevent him from punishing them for expropriating commoners' assets, and the expected income from such expropriations from commoners may be counted as part of the officials' compensation. In such a situation, as our model assumes, the leader would be able to guarantee large debts H to the elite members of his inner circle, while others in the population could not hold assets worth more than K without serious risk of expropriation, with $K < G < H$.

10. Conclusions

We have considered an extension of the Becker-Stigler and Shapiro-Stiglitz dynamic agency models to emphasize the crucial problems of judging and punishing high officials of the state. (See also Biais, Mariotti, Rochet, Villeneuve, 2007.) For high officials to be deterred from abusing their power, they must be confident that loyal service will bring great expected rewards. Indeed, we have seen that an inability of the state to credibly promise sufficiently great rewards for good service may sometimes cause the state to demand less of its agents when their credit is low, forgiving costly losses that are evidence of low effort or corruption, as in Kornai's soft budget constraint. When the stakes are high enough to justify a system of hard budget constraints, some punishment of loyal hard-working officials may be unavoidable because monitoring is imperfect. But such punishments raise a fundamental problem of trust at the center of the state because, ex post, penalties reduce the debt of the state, and dismissals can become profitable opportunities for the head of state to resell valuable offices. The need to apply a randomized punishment strategy, to deter rebellions, requires that the process of judging high officials must be monitored in detail. Thus, we have argued that a political leader needs to guarantee the credibility of his incentive plan for high officials by a constitutional system in which high officials cannot be punished without a trial that is witnessed by others in an institutionalized court or council.

When these institutionalized protections apply only to the small governing elite, so that people outside this elite cannot accumulate the wealth that high officials must be guaranteed, then the leader's optimal incentive plan should minimize the expected turnover of high officials. Such minimization of turnover can be accomplished by deferring compensation and increasing the leader's debt to his officials, so that most crises can be penalized by temporarily suspending some payments on this debt without actually dismissing the responsible official. Thus, moral-hazard problems can provide a positive incentive for political leaders to accumulate the largest possible debts to their officials, even when leaders and officials discount the future at the same rate. In the long run, however, this accumulation of such debts will create an elite aristocracy that holds large expensive claims on the state.

Such accumulated debts of aristocratic privileges can ultimately weaken the state by decreasing the resources that it can apply to defend itself against other challenges. But these institutionalized debts can be repudiated only when a new dynasty begins with a leader who is not bound by any inherited promises of predecessors to their governing elite. Thus, our model may offer some explanation of traditional dynastic declines in history.

Notice, however, that the rationale here for vesting high offices in the members of an elite aristocracy is not based on any assumption of innate inequality among individuals. Indeed, our analysis implicitly assumed an innate equality among individuals, because we have ignored the possibility that a policy of recruiting broadly from the mass of poor commoners might yield more capable officials than the leader can find among his trusted inner circle of aristocrats. In our analysis, elitism results instead from a scarcity of social trust: trust by leaders that their agents will not abuse delegated powers for short-term private gain, and trust by agents that the leader will actually pay deferred rewards after long years of loyal service.

In essence, the expected rewards that must be associated with high offices make them valuable assets, for which qualified candidates would be willing to pay ex ante, to the extent that they have the means. But these expected rewards are also a liability of the state, and the leader of the state would profit ex post by repudiating such liabilities. So high officials must be recognized as having acquired valuable rights to their offices, and these rights require institutional protection and legal enforcement, like any other property rights. People generally look to the state for enforcement of property rights, but in this case we are considering the enforcement of debts owed by the leader of the state himself. The key to enforcement in this case may be the observation that the leader cannot get loyal service from his high officials if they lose their trust that he will fulfill his promises. If high officials have a shared sense of identity, then the wrongful dismissal of any one of them can cause others to lose trust in the leader. So when there is any question about whether a high official has been wrongly dismissed by the leader, the jury for deciding this question can be found among the other high officials. Thus, the leader's essential credibility can be maintained by instituting a court or council where any judgment against a high official will be scrutinized by others in their privileged class.

We began by noting a few historical examples of institutions that have served the essential function of regulating the punishment of powerful government agents: the Hittite *Panku* council, the Roman Senate, and the English Exchequer. The importance of the medieval Exchequer can be particularly appreciated when we contrast it with the centrifugal forces of feudalism in the same period. The king's ability to exercise authority throughout the realm depended critically on the sheriffs' confidence that loyal service would earn the king's reward, but a policy of never punishing sheriffs would constitute surrender to feudal disintegration of the state. Strong centralized government required credible guarantees that punishment of local government agents would occur only in carefully limited circumstances situations, and such guarantees required formal institutions to protect the agents' rights. From this perspective, we can also see how the subsequent development of parliamentary representation in England (from the thirteenth century) could further strengthen royal government, by enabling kings to credibly guarantee privileges to a broader class of local government agents than could be practically assembled at court (Coss, 2005, ch 7).

In the absence of such institutional guarantees, when governors and local commanders lose trust that their ruler will treat them fairly, the results can be disastrous for the state, as the Hittite king Telipinu warned 3500 years ago. After 193, the Roman emperor Septimus Severus began promoting generals from lower-class origins, but the Roman Senate as an institution could effectively guarantee fair trials only for members of the elite senatorial aristocracy. So this meritocratic policy may explain the increasing frequency of military rebellions that plagued the Roman Empire after Septimus Severus. The ultimate fall of the Western Roman Empire followed after the treacherous murder of the Roman general Aëtius by the emperor Valentinian III (Grant, 1985). Similarly, the collapse of the Ming dynasty in China began in 1630 when the Ming emperor was seen to unjustly execute his commander against the Manchus, Yuan Chonghuan, whose string of celebrated victories had been broken by one reversal. Thereafter, other Chinese commanders regularly defected to the Manchus, who soon replaced the Ming as rulers of China (Mote, 1999, ch 30). The moral-hazard constraints in our model are a simple stylized representation of the standards of expected behavior that a leader must satisfy to avoid such disasters and maintain the state.

11. Appendix (proofs)

Theorem 1. The optimality conditions (9)-(13) are satisfied by the incentive plan described above in equations (14)-(18). So the value function V satisfies (19)-(21).

Proof. Because V is nondecreasing, the minimum in (10) is achieved at $g=G$, yielding equation (14), $V(0) = V(G)-K$.

When $G\leq u<H$, the minimization in (11) tells us that the optimal policy should choose $\pi\geq\tau$ and $y\geq 0$ to minimize $y+(\delta u+\alpha\pi-y)V'(u)+\alpha V(u-\pi)-\alpha V(u)$. This formula is increasing in y

because the slope V' is between 0 and 1, and it is increasing in π because V is convex. So when $G \le u < H$, the optimal policy must have $\pi=\tau$ and $y=0$, as in (15)-(16). Similarly, when u=H, the minimization in (12) tells us that the optimal policy should choose $\pi \ge \tau$ and $\theta \le 0$ to minimize $\delta H+\alpha\pi-\theta+\theta V'(H)+\alpha V(H-\pi)-\alpha V(H)$. Because $V' \le 1$, this minimum for u=H is achieved by setting $\theta=0$ and $\pi=\tau$, as in (15) and (17).

Substituting these optimal policies from (15) and (16) into equation (11) yields equation (20) in the theorem. Similarly, at the upper bound H, equation (12) with (15) and (17) becomes equation (21) in the theorem. With (20) and (21), continuity of the convex function V implies

$$\lim_{u \to H} V'(u) = 1.$$

Now given any $\hat{u}$ in [0,G), we need to find optimum for (13). We must have the binding constraint $Y+(1-q)g = \hat{u}$, because otherwise Y could be decreased or q increased. Substituting $1-q = (\hat{u}-Y)/g$, condition (13) becomes

$$V(\hat{u}) = \min_{Y,g}\ Y+V(0)+(V(g)-V(0))(\hat{u}-Y)/g \ \text{ subject to }\ Y \ge 0,\ g \ge G.$$

Then $V' \le 1$ implies $[V(g)-V(0)]/g \le 1$, and so this minimum is achieved with Y = 0. So

$$V(\hat{u}) = \min_{g}\ V(0)+(V(g)-V(0))\hat{u}/g \ \text{ subject to }\ g \ge G.$$

But $\partial/\partial g\ [(V(g)-V(0))\hat{u}/g] = [V(0) - (V(g)-gV'(g))]/g^2 \ge 0$, because V is convex. So the minimum is achieved with $Y=0$, $g=G$, $q = 1-\hat{u}/G$, as (18) asserts, and

$$V(\hat{u}) = V(0)+(V(G)-V(0))\hat{u}/G.$$

Then with (14), we get equation (19), $V(\hat{u}) = V(0)+\hat{u}K/G$.

By (19), the slope of $V(\hat{u})$ is K/G for all $\hat{u}$ between 0 and G. There is a kink at G, and the formula (20) at u=G is actually the slope on the high side, for increases of credit above G.

It remains to show that these equations (19)-(21) characterize a function V which is actually convex, as condition (9) requires. We do so now by showing how to compute V numerically. The trick is to consider the function Ψ such that

$$\Psi(u) = (V(u) - uK/G)/V(0),\ \ \forall u \in [0,H].$$

With equations (19)-(20), this function Ψ satisfies the equations

(22) if $u \le G$ then $\Psi(u) = 1$,

(23) if $u \ge G$ then $\Psi'(u) = [(\delta+\alpha)\Psi(u) - \alpha\Psi(u-\tau)]/(\delta u+\alpha\tau)$.

(To verify (23), notice that the differential equation in (20) is linear in V, and it would also be satisfied by the linear function uK/G.) This function $\Psi(u)$ is continuous, but its derivative $\Psi'(u)$

has one discontinuity at u=G, where equation (23) yields the right derivative $\Psi'(G) = \delta/(\delta G+\alpha\tau)$, which indicates how a small increase of u above G would increase $\Psi(u)$. Then the differential equation (23) can be solved numerically to compute $\Psi(u)$ for all $u \geq G$, and V(u) can be computed from $\Psi(u)$. Lemma 1 tells us that this construction gives V the convexity and slope properties that were specified by condition (9), as required to complete the proof of Theorem 1.

Lemma 1. An increasing convex function Ψ can be uniquely characterized by conditions (22) and (23). This function $\Psi(u)$ is continuous, but its derivative $\Psi'(u)$ has one discontinuity at u=G, where equation (23) yields the right derivative. When $u \geq G$, the derivative $\Psi'(u)$ is strictly increasing in u. Then prince's optimal value function V can be computed from this function Ψ by the equations $V(0) = (1-K/G)/\Psi'(H)$, and $V(u) = uK/G + \Psi(u)V(0)$, $\forall u \in [0,H]$.

Proof. We have seen that the derivative Ψ' increases discontinuously at G, from $\Psi'(u)=0$ when $u<G$ to $\Psi'(G) = \delta/(\delta G+\alpha\tau) > 0$. At any $u \geq G$, differentiating (23) yields

$$\begin{aligned}\Psi''(u) &= [(\delta+\alpha)\Psi'(u) - \alpha\Psi'(u-\tau)]/(\delta u+\alpha\tau) \\ &\quad - [(\delta+\alpha)\Psi(u) - \alpha\Psi(u-\tau)]\delta/(\delta u+\alpha\tau)^2 \\ &= [(\delta+\alpha)\Psi'(u) - \alpha\Psi'(u-\tau) - \delta\Psi'(u)]/(\delta u+\alpha\tau) \\ &= \alpha[\Psi'(u) - \Psi'(u-\tau)]/(\delta u+\alpha\tau).\end{aligned}$$

This formula yields $\Psi''(G)>0$, because $\Psi'(G) = \delta/(\delta G+\alpha\tau) > \Psi'(G-\tau) = 0$. If $\Psi''(u)$ were not always strictly positive when $u \geq G$, then there would exist some smallest $\hat{u} \geq G$ such that $\Psi''(\hat{u})=0$; but then $\Psi''(u)>0$ for all smaller u would imply $\Psi'(\hat{u}) > \Psi'(\hat{u}-\tau)$, yielding the contradiction

$$\Psi''(\hat{u}) = \alpha[\Psi'(\hat{u})-\Psi'(\hat{u}-\tau)]/(\delta\hat{u}+\alpha\tau) > 0.$$

So $\Psi''(u) > 0$ for all $u \geq G$. Thus, $\Psi'(u)$ is strictly increasing and positive when $u \geq G$, and $\Psi(u)$ is a convex function of u.

But the initial conditions and differential equation (22)-(23) that uniquely characterize $\Psi(u)$ are conditions that are satisfied by $(V(u)-uK/G)/V(0)$, and so we get

$$V(u) = uK/G + \Psi(u)V(0), \quad \forall u \in [0,H].$$

So convexity of $\Psi(u)$ implies convexity of V(u). Differentiation yields

$$V'(u) = K/G + \Psi'(u)V(0).$$

To compute V(u) from $\Psi(u)$, we need to evaluate V(0). But from (20)-(21) we have

$$V'(H)=1, \text{ and so } K/G + \Psi'(H)V(0) = 1.$$

Thus, $V(0)$ can be computed from Ψ by $V(0) = (1-K/G)/\Psi'(H)$. Q.E.D.

Theorem 3. The stationary strict cumulative distribution F satisfies, for any integer $m \geq 0$,

$$\text{if } u \leq H-m\tau \text{ then } F(u) \leq [\alpha\tau/(\delta G+\alpha\tau)]^{m+1}.$$

When credit **u** is drawn from the stationary distribution F, its expected value satisfies

$$E(\mathbf{u}) \geq H-\alpha\tau^2/(\delta G),$$

and its probability of being at the upper bound H satisfies

$$P(\mathbf{u}=H) = 1 - F(H) \geq (\delta H - \alpha\tau^2/G)/(\delta H+\alpha\tau).$$

Proof. We first prove the following equation, for any w in the interval [G,H]:

$$\int_{u\in[G,w)} \delta u \, dF(u) + \int_{u\in[w,w+\tau]} \alpha(u-w-\tau) \, dF(u) + \int_{u\in[G,G+\tau]} \alpha(G+\tau-u) \, dF(u) = 0. \quad (29)$$

To prove (29) for any w, consider the quantity

$$E(\min\{0,\mathbf{u}-w\}) = \int_{u\in[G,w]} (u-w) \, dF(u).$$

With the stationary distribution F, this expected value is constant over time. The left-hand side of (29) is the rate at which this expected value would increase over time, starting with the distribution F. The first integral in (29) is $F(w)\,E(\delta\mathbf{u} \mid \mathbf{u}<w)$, which is the expected rate of increase of the current governor's credit **u** when starting from a state with $\mathbf{u}<w$. The second integral is the rate at which newly nonzero (negative) values of $\min\{0, \mathbf{u}-w\}$ are created from a governor with credit above w having a crisis that brings her credit below w. When the governor's credit drops below G, credit is raised to G by either reinstatement or replacement, and the rate of such raises is $\alpha E(\max\{G+\tau-\mathbf{u}, 0\})$, the third integral.

Notice that $\delta G \leq E(\delta\mathbf{u} \mid \mathbf{u}<w)$, and $\int_{u\in[w,w+\tau]} \alpha(w+\tau-u) \, dF(u) \leq \alpha\tau[F(w+\tau)-F(w)]$, and the third integral in equation (29) is nonnegative. Together, these inequalities imply

$$F(w)\,\delta G \leq F(w)\,E(\delta\mathbf{u} \mid \mathbf{u}<w) \leq \int_{u\in[w,w+\tau]} \alpha(w+\tau-u) \, dF(u) \leq \alpha\tau[F(w+\tau)-F(w)], \quad (30)$$

and thus,

$$F(w) \leq F(w+\tau)\,\alpha\tau/(\delta G+\alpha\tau), \quad \forall w\in[G,H].$$

Notice $F(w+\tau) = 1$ when $w+\tau > H$. So by induction, for any integer $m \geq 0$,

$$\text{if } u \leq H-m\tau \text{ then } F(u) \leq [\alpha\tau/(\delta G+\alpha\tau)]^{m+1}.$$

These bounds tell us that the steady-state credit **u** is unlikely to be far below H, satisfying

$$E(H-\mathbf{u}) = \int_{u\in[G,H]} (H-u) \, dF(u) = \int_{u\in[G,H]} F(u) \, du$$

$$\leq \sum_{m=0}^{\infty} \tau\,[\alpha\tau/(\delta G+\alpha\tau)]^{m+1} = \alpha\tau^2/(\delta G).$$

Thus $H-\alpha\tau^2/(\delta G) \le E(\mathbf{u})$. Using (30) with w=H and using F(H+τ)=1, we get

$$E(\delta\mathbf{u}) = F(H)\, E(\delta\mathbf{u} \mid \mathbf{u}<H) + [1-F(H)]\delta H \le \alpha\tau[1-F(H)] + [1-F(H)]\delta H.$$

Thus $\delta H-\alpha\tau^2/G \le E(\delta\mathbf{u}) \le [1-F(H)](\delta H+\alpha\tau)$ and so we get

$$(\delta H-\alpha\tau^2/G)/(\delta H+\alpha\tau) \le 1-F(H). \qquad \text{Q.E.D.}$$

Theorem 4. When low effort can be tolerated, the optimal incentive plan is characterized by one of two cases. When $W(0) \le \beta L/\delta$, the HBC case applies: a new governor starts at credit $G=D+\tau$, and high effort is always demanded according to the optimal plan in Theorem 1, and

$$W(u) = V(u)+(\alpha L+\gamma)/\delta, \ \forall u\in[0,H].$$

When $W(0) > \beta L/\delta$, the SBC case applies: a new governor starts at credit D, and the optimal policy tolerates low effort with no crisis penalties while the governor's credit u satisfies $D\le u<G$, but the optimal policy demands high effort when $u\ge G$, and so the expected cost function satisfies $W'(u) = \Omega_0(u)$ when $D\le u<G$, but $W'(u) = \Omega_1(u)$ when $G\le u\le H$. In the SBC case, costs are

$$W(u) = u + (V(u)-u)(\beta L-\alpha L-\gamma)/(\delta V(0)) + (\alpha L+\gamma)/\delta, \ \forall u\ge D,$$

and $W(0)=W(D)$. The SBC case applies when $V(0) > (\beta L-\alpha L-\gamma)/\delta$.

Proof. For u<D, both the Ω_0 and Ω1 are zero, so the differential equation (28) is $W'(u)=0$, and so $W(u) = W(0), \ \forall u\le D$.

At u=D, $\Omega_0(u)$ jumps discontinuously from 0 to $\Omega_0(u) = (W(u) - \beta L/\delta)/u$. So now let us consider $u\ge D$.

In any interval where low effort is optimal and so $W'=\Omega_0$, the value function W(u) becomes linear in u; that is,

$$\text{if } W'(u) = \Omega_0(u) = (W(u) - \beta L/\delta)/u \text{ then } W''(u) = \Omega_0'(u) = 0.$$

To verify this linearity result, notice that, when $u\ge D$,

$$\Omega_0'(u) = [uW'(u)-(W(u)-\beta L/\delta)]/u^2 = [W'(u)-\Omega_0(u)]/u$$

which is equal to zero when $W'(u) = \Omega_0(u)$.

So for any credit u in the interval from D to G, where a governor can be asked to exert low effort but not high effort, the differential equation (27) becomes

$$W'(u) = \max\{0, \Omega_0(u)\} \text{ when } D \le u < G.$$

In this maximum, the first term 0 applies when the new governor is promoted directly to credit G, and the second term applies when the new governor starts at credit D and exerts low effort until her credit reaches the minimal level where high effort can be motivated without inciting rebellion. Notice that $\Omega_0(D) = (W(D)-\beta L/\delta)/D$. So $\Omega_0(D)$ is greater or less than 0 depending on whether $W(0)$ is greater or less than $\beta L/\delta$.

If $W(0) \le \beta L/\delta$ then $\max\{0,\Omega_0(D)\} = 0$, and then the differential equation (27) becomes $W'(u) = 0$ and $W(u) = W(0)$ for all $u\in[0,G]$.

On the other hand, if $W(0) > \beta L/\delta$, then $W'(D) = \max\{0,\Omega_0(D)\} = \Omega_0(D)$ and so, by the constancy of Ω_0 when W' equals Ω_0, the expected cost function $W(u)$ must be linear in u on the interval from D to G, with $W'(u) = \Omega_0(D) = (W(D)-\beta L/\delta)/D$. So if $W(0) > \beta L/\delta$ then

$$W(u) = W(0)+(u-D)(W(0)-\beta L/\delta)/D, \ \forall u\in[D,G).$$

Now consider $u\ge G$, so that high effort is feasible. At $u=G$, $\Omega_1(u)$ jumps discontinuously from 0 to $\Omega_1(u) = [\delta W(u)+\alpha W(u)-\alpha W(u-\tau)-\alpha L-\gamma]/(\delta u+\alpha\tau)$. With $u\ge G$, we get

$$(\delta u+\alpha\tau)[\Omega_1(u) - \Omega_0(u)] = \alpha[W(u) - W(u-\tau) - \tau\Omega_0(u)] + \beta L-\alpha L-\gamma.$$

At $u=G=D+\tau$, we have

$$W(G)-W(G-\tau) = \tau\max\{0,\Omega_0(G)\} \ge \tau\Omega_0(G)$$

which, with the given parametric inequality $\beta L>\alpha L+\gamma$, implies that

$$\Omega_1(G) > \Omega_0(G).$$

Thus, the governor should optimally use high effort when her credit is G.

Furthermore, the high effort must remain optimal at all higher credit levels. To verify this claim, notice that a switch to low effort above some credit $u>G$ would make W' equal to Ω_0 at such credit levels, but then $\Omega_0'(u)$ would become 0, and so we would get

$$d/du\ [(\delta u+\alpha\tau)(\Omega_1(u) - \Omega_0(u))] = \alpha[W'(u) - W'(u-\tau)] > 0,$$

which would make it impossible for $\Omega_1(u)-\Omega_0(u)$ to become negative as credit u increases.

In the SBC case when $W(0) > \beta L/\delta$, W is determined from $W(0)$ by the equations

$$W(u) = W(0)+(u-D)(W(0)-\beta L/\delta)/D, \ \forall u\in[D,G],$$

$$W'(u) = [\delta W(u)+\alpha W(u)-\alpha W(u-\tau)-\alpha L-\gamma]/(\delta u+\alpha\tau), \ \forall u\in[G,H].$$

With the function Ψ from Lemma 1, these equations are uniquely satisfied by

$$W(u) = u(W(0)-\beta L/\delta)/D + (\alpha L+\gamma)/\delta + \Psi(u)(\beta L-\alpha L-\gamma)/\delta.$$

Then equation (28) gives us

$$1 = W'(H) = (W(0)-\beta L/\delta)/D + \Psi'(H)(\beta L-\alpha L-\gamma)/\delta.$$

By Lemma 1, $\Psi(u) = V(u)/V(0)$, and so we get

$$W(0) = \beta L/\delta + D[1-(\beta L-\alpha L-\gamma)/(\delta V(0))],$$

$$W(u) = u + (V(u)-u)(\beta L-\alpha L-\gamma)/(\delta V(0)) + (\alpha L+\gamma)/\delta,\ \ \forall u \geq D.$$

In the HBC case, the expected total cost here is $W(u) = V(u)+(\alpha L+\gamma)/\delta$, that is, the expected discounted wage cost V(u) from Theorem 1 plus the expected cost of crises and maintainance. But V(u)>u, because the expected wage cost is greater than the expected value of the current governor's wages, and so the SBC formula for W(u) is less than the HBC formula when $(\beta L-\alpha L-\gamma)/(\delta V(0)) < 1$. Q.E.D.

Finally, we can describe how the solution in Theorem 4 would change with K>0. Let $V_0(u)$ denote the value of V(u) in Theorem 1 with K=0 but with all other parameters are as given; that is $V_0(u)$ is the expected total cost of wages under the HBC plan. For the SBC plan, expected net cost with K>0 would be the same as with K=0 once a governor has been appointed, and so

$$W_{SBC}(u) = u + (V_0(u)-u)(\beta L-\alpha L-\gamma)/(\delta V_0(0)) + (\alpha L+\gamma)/\delta,\ \ \forall u \geq D,$$

but now $W_{SBC}(0) = W_{SBC}(D) - K$. The high-effort HBC plan has a new wrinkle, however. When we allow that a governor can trust the prince even with credit as low as D, then a new governor can be asked to pay K for appointment with credit D, and then, after a short period of time, the new governor can be either promoted to credit G, with probability D/G, or else dismissed. High effort cannot be demanded until the governor is promoted to G, but this low-effort period could be made arbitrarily short. With such randomization for new appointments, any dismissal would be quickly followed by the prince collecting K from a random number of new governors that has expected value G/D. So we can implement a modified HBC plan that differs from the optimal policy in Theorem 1 only in that, after any crisis-penalty dismissal, the prince can resell the governor's office for expected revenue KG/D instead of K, and so (applying Lemma 1) the expected net cost becomes

$$W_{HBC}(u) = u + (V_0(u)-u)(1-K/D) + (\alpha L+\gamma)/\delta,\ \ \forall u \geq 0.$$

The SBC plan is optimal when $(\beta L-\alpha L-\gamma)/(\delta V_0(0)) < (1-K/D)$, and otherwise the modified HBC plan is optimal.

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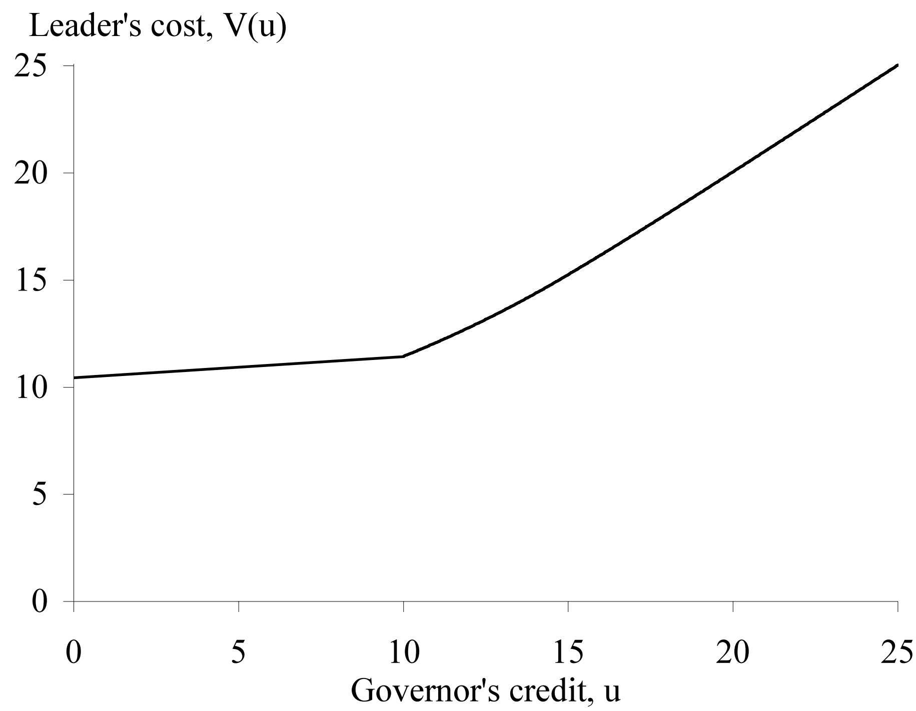

Figure 1. The leader's expected cost as a function of the credit owed to the governor, for an example with α=0.1, β=0.3, δ=0.05, γ=1, D=5, K=1, H=25.

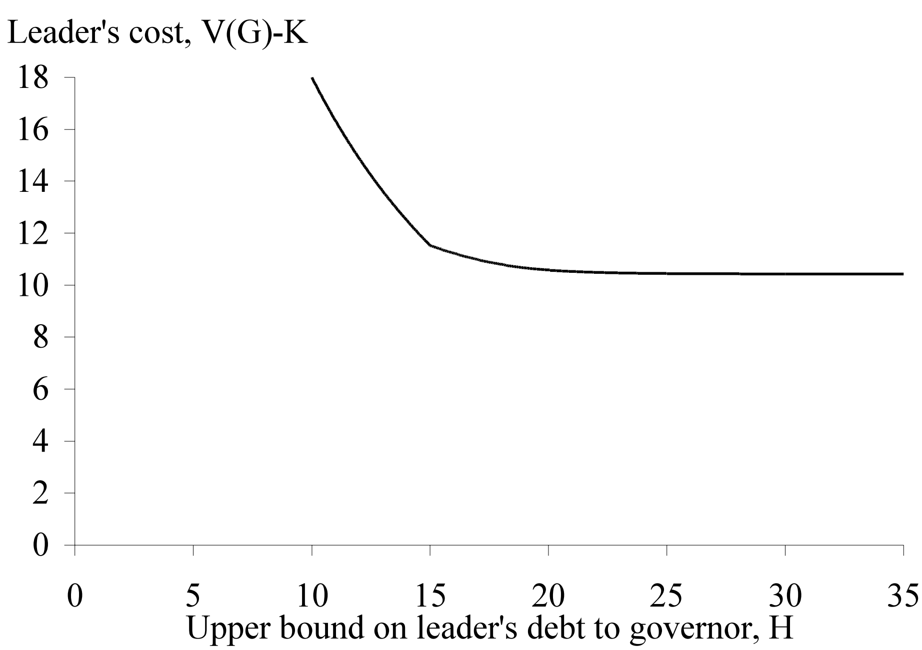

Figure 2. The leader's expected cost when appointing a new governor, as a function of the credit bound H, with α=0.1, β=0.3, γ=1, D=5, K=1.

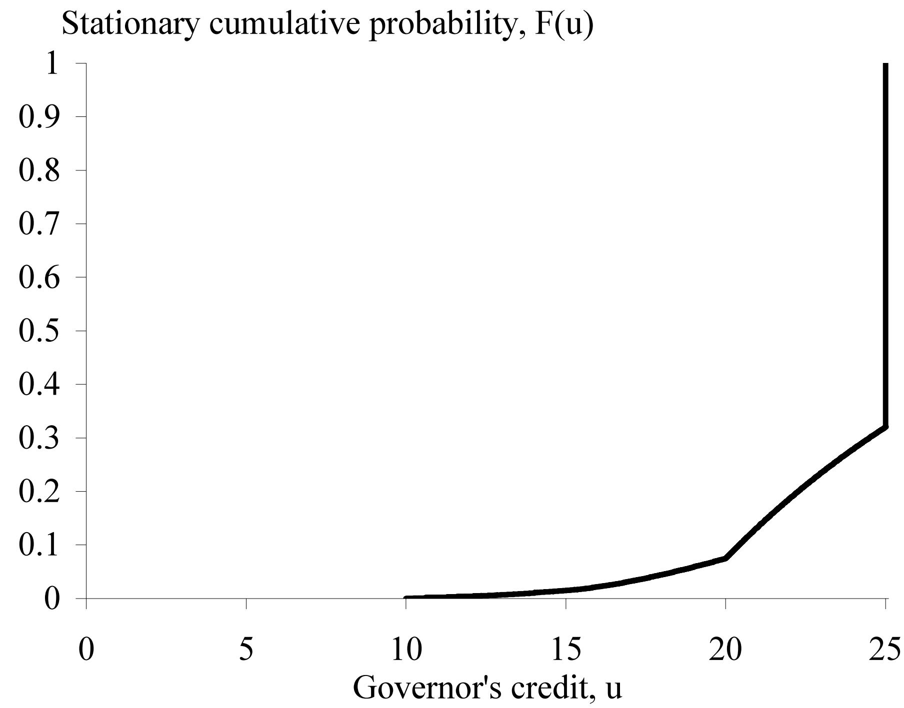

Figure 3. The stationary probability distribution of governors' credit, with α=0.1, β=0.3, δ=0.05, γ=1, D=5, K=1, H=25.

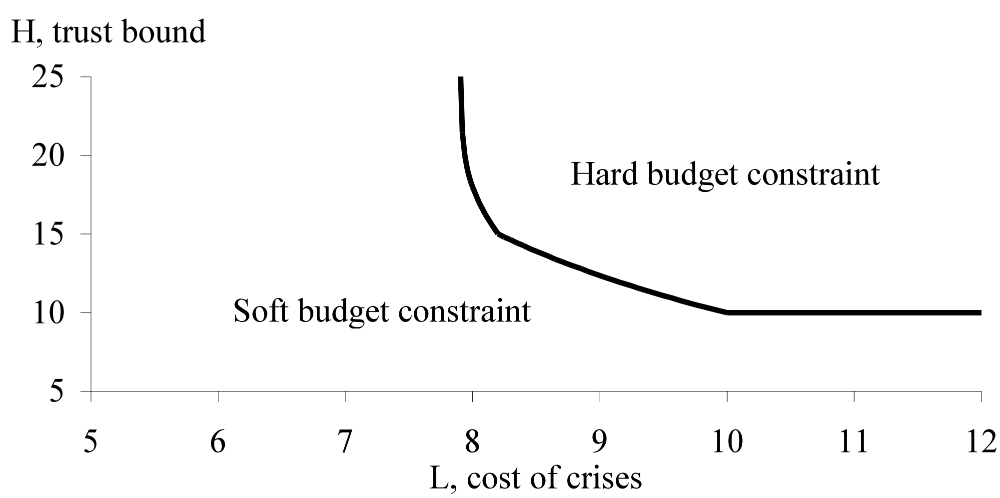

Figure 4. Optimality of incentive plans with hard or soft budget constraints, with α=0.1, β=0.3, γ=1, δ=0.05, D=5, K=0.