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"Global Warming, Proportional Carbon Taxes, and International Fund
for Atmospheric Stabilization"

H. Uzawa, Social Common Capital Research, Tokyo, Japan

Introduction

The atmospheric concentration of greenhouse gases, particularly carbon dioxide, has been increasing since the times of the Industrial Revolution, with an accelerated rate in the last several decades. According to the IPCC reports, it is estimated that, if the emission of carbon dioxide and other greenhouse gases and the disruption of tropical rain forests were to continue at the present pace, global average air surface temperature toward the end of the twenty-first century would be 3–6 °C higher than the level prevailing before the Industrial Revolution, resulting in drastic changes in climatic conditions and accompanying disruption of the biological and ecological environments.

The problems of global warming are genuinely dynamic. From past human activities we inherit an excess concentration of atmospheric CO₂, and the choices we make today concerning the use of fossil fuels and related activities significantly affect all future generations through the phenomenon of global warming that is brought about by the atmospheric concentrations of CO₂ due to the combustion of fossil fuels today. Thus, we

explicitly have to take into account the decreases in the welfare levels of all future generations caused by the increases in the atmospheric accumulations of CO₂ today.

The International Fund for Atmospheric Stabilization is an institutional framework in which it is possible to combine an international arrangement to stabilize atmospheric equilibrium with a redistributive scheme to help developing countries to accelerate processes of economic development.

The Model

We denote by V_t the amount of CO₂ that has accumulated in the atmosphere at time t . The quantity V_t is measured in actual tons (in weights of carbon content) or in terms of the density of CO₂ in the atmosphere. We also adopt as the origin of the measurement the stable pre-Industrial Revolution level of 600 GtC (=10⁹ tons in carbon content), approximately corresponding to the density of 280 ppm (= part per million). The current level of 760 GtC (approximately 380 ppm) is expressed as $V_t = 160$ GtC.

The atmospheric accumulations of CO₂ change over time as the result of natural and anthropogenic factors. A certain portion of atmospheric concentrations of CO₂ is absorbed by the oceans (roughly estimated at 50%) and to a lesser extent by living terrestrial plants.

Approximately 75-90 GtC of carbon are annually exchanged between the atmosphere and the surface oceans. We assume that the amount of atmospheric CO₂ annually absorbed by the oceans is given by μV_t , where V_t is the atmospheric concentrations of CO₂ measured

in actual tons of CO₂ with the pre-Industrial Revolution level of 600 GtC as the origin of measurement. The rate of absorption μ would have a magnitude 2—4%.

In the simple dynamic model, we assume that the anthropogenic change in atmospheric CO₂ is exclusively caused by the combustion of fossil fuels in connection with industrial, agricultural, and urban activities.

The change in the atmospheric level of CO₂ is given by

$$\dot{V}_t = a_t - \mu V_t,$$

where a_t is the annual rate of increase in the atmospheric level of CO₂ due to anthropogenic activities and μV_t is the amount of atmospheric CO₂ annually absorbed by the oceans.

The rate of anthropogenic change in the atmospheric level of CO₂, a_t , is determined by the combustion of fossil fuels and is closely related to the levels of production and consumption activities conducted during the year observed.

There are a finite number of individual countries in the world that share the earth's atmosphere as social common capital. Each country is generically denoted by $v = 1, \dots, n$.

We assume that the utility function for each country v is expressed in the following manner:

$$u^v = u^v(c^v, a_c^v, V),$$

where $c^v = (c_j^v)$ is the vector of goods consumed in country v on the per capita basis, a_c^v is the amount of CO₂ emissions released during the processes of consumption, and V is the atmospheric concentration.

For each country v , we also assume that utility function $u^v(c^v, a_c^v, V)$ is strongly separable with respect to (c^v, a_c^v) and V :

$$u^v = \phi^v(V) u^v(c^v, a_c^v).$$

The function $\phi^v(V)$ expresses the extent to which people in country v are adversely affected by global warming. It is referred to as the impact index of global warming. We assume that the impact index function $\phi^v(V)$ of global warming for each country v satisfies the following conditions:

$$\phi^v(V) > 0, \phi^{v'}(V) < 0, \phi^{v''}(V) < 0 \text{ for all } V > 0.$$

The *impact coefficient* of global warming for country v is the relative rate of the marginal change in the impact index due to the marginal increase in the atmospheric accumulation of CO₂; that is,

$$\tau^v(V) = -\frac{\phi^{v'}(V)}{\phi^v(V)}.$$

We assume that the impact coefficients of global warming $\tau^v(V)$ are identical for all countries v :

$$\tau^v(V) = \tau(V) \text{ for all } v.$$

The impact coefficient function $\tau(V)$ satisfies the following conditions:

$$\tau(V) > 0, \tau'(V) > 0 \text{ for all } V > 0.$$

The impact index function $\phi(V)$ of the following form is often postulated:

$$\phi(V) = (\hat{V} - V)^\beta, \quad 0 < V < \hat{V},$$

where \hat{V} is the critical level of the atmospheric accumulation of CO_2 and β is the sensitivity parameter ($0 < \beta < 1$). The critical level \hat{V} of the atmospheric accumulation of CO_2 is usually assumed to be twice the level prevailing before the Industrial Revolution; that is, $\hat{V} = 600$ GtC. The impact coefficient $\tau(V)$ is given by

$$\tau(V) = \frac{\beta}{\hat{V} - V}.$$

Utility functions $u^v(c^v, a_c^v)$ satisfy the following neoclassical conditions:

(U1) Utility function $u^v(c^v, a_c^v)$ is defined, positive, continuous, and continuously twice-differentiable for all $(c^v, a_c^v) \geq 0$.

(U2) Marginal utilities are positive both for the consumption of goods c^v and CO_2 emissions a_c^v :

$$u_{c^v}^v(c^v, a_c^v) \geq 0, u_{a_c^v}^v(c^v, a_c^v) \geq 0 \quad \text{for all } (c^v, a_c^v) \geq 0.$$

(U3) Utility function $u^v(c^v, a_c^v)$ is strictly quasi-concave with respect to $(c^v, a_c^v) \geq 0$.

(U4) Utility function $u^v(c^v, a_c^v)$ is homogeneous of order 1 with respect to c^v :

$$u^v(tc^v, ta_c^v) = tu^v(c^v, a_c^v) \quad \text{for all } (c^v, a_c^v) \geq 0.$$

The Consumer Optimum

The world markets for produced goods are assumed to be perfectly competitive and prices of goods are denoted by a nonzero, nonnegative vector p ($p \geq 0$). Carbon taxes at the rate θ^v

are levied upon the emission of CO₂ in each country ν . Suppose national income of country ν in units of world prices is given by y^ν . Then, the consumers in country ν would choose consumption vector c^ν and CO₂ emissions a_c^ν that maximize country ν 's utility function

$$u^\nu(c^\nu, V) = \phi^\nu(V) u^\nu(c^\nu, a_c^\nu)$$

subject to the budget constraints

$$pc^\nu + \theta^\nu a_c^\nu = y^\nu, \quad (c^\nu, a_c^\nu) \geq 0.$$

We have the following relation:

$$\alpha^\nu \phi^\nu(V) u^\nu(c^\nu, a_c^\nu) = y^\nu,$$

where α^ν is the inverse of the marginal utility of income y^ν of country ν .

Specifications for Production Possibility Sets

The conditions concerning the production of goods in each country ν are specified by the production possibility set T^ν that summarizes the technological possibilities and organizational arrangements for country ν ; the endowments of factors of production available in country ν are given.

We assume that there are a finite number of factors of production that are essentially needed in the production of goods. They are generically denoted by f ($f = 1, \dots, F$).

The endowments of factors of production available in each country ν are expressed by a vector $K^\nu = (K_f^\nu)$, where $K^\nu \geq 0$.

In each country ν , the minimum quantities of factors of production required to produce goods by the vector of production x^ν with emissions at the level a_p^ν are specified by a vector-valued function:

$$f^\nu(x^\nu, a_p^\nu) = (f_f^\nu(x^\nu, a_p^\nu)).$$

We assume that

(T1) $f^\nu(x^\nu, a_p^\nu)$ are defined, positive, continuous, and continuously twice-differentiable for

all $(x^\nu, a_p^\nu) \geq 0$.

(T2) $f_{x^\nu}^\nu(x^\nu, a_p^\nu) > 0, f_{a_p^\nu}^\nu(x^\nu, a_p^\nu) \leq 0$ for all $(x^\nu, a_p^\nu) \geq 0$.

(T3) $f^\nu(x^\nu, a_p^\nu)$ are strictly quasi-convex with respect to (x^ν, a_p^ν) for all $(x^\nu, a_p^\nu) \geq 0$.

(T4) $f^\nu(x^\nu, a_p^\nu)$ are homogeneous of order 1 with respect to (x^ν, a_p^ν) for all $(x^\nu, a_p^\nu) \geq 0$.

The production possibility set of each country ν , T^ν , is composed of all combinations (x^ν, a_p^ν) of vectors of production x^ν and CO₂ emissions a_p^ν that are possibly produced with the organizational arrangements and technological conditions in country ν and the given endowments of factors of production K^ν of country ν . Hence, it may be expressed as

$$T^\nu = \{(x^\nu, a_p^\nu) : (x^\nu, a_p^\nu) \geq 0, f^\nu(x^\nu, a_p^\nu) \leq K^\nu\}.$$

Postulates (T1—3) imply that the production possibility set T^ν is a closed, convex set of $J+1$ -dimensional vectors (x^ν, a_p^ν) .

The Producer Optimum

The producers in country v would choose those combinations (x^v, a_p^v) of vectors of production x^v and CO₂ emissions a^v that maximize net profits

$$px^v - \theta^v a_p^v$$

over $(x^v, a_p^v) \in T^v$.

Conditions (T1–3) postulated above ensure that, for any combination of price vector p and carbon tax rate θ^v , the optimum combination (x^v, a_p^v) of vector of production x^v and emissions a_p^v always exists and is uniquely determined.

Market Equilibrium and Global Warming

Suppose that carbon taxes at the rate of θ^v are levied on the emission of in each country v .

Market equilibrium for the world economy is obtained if we find the prices of goods at which total demand is equal to total supply:

$$\sum_v c^v = \sum_v x^v.$$

Total emissions a are given by

$$a = \sum_v a^v, \quad a^v = a_c^v + a_p^v.$$

(i) Demand conditions in each country v are obtained by maximizing utility function

$$u^v = \phi^v(V) u^v(c^v, a_c^v)$$

subject to budget constraints

$$pc^v + \theta^v a_c^v = y^v,$$

where y^v is the national income of country v .

(ii) Supply conditions in each country v are obtained by maximizing net profits

$$px^v - \theta^v a_p^v$$

over $(x^v, a_p^v) \in T^v$.

(iii) Total emissions in the world, a , are given as the sum of emissions in all countries; that

is,

$$a = \sum_v a^v, \quad a^v = a_c^v + a_p^v.$$

Imputed Price and Sustainable Development

In the dynamic analysis of global warming, a crucial role is played by the concept of the imputed price ψ_t^v of the atmospheric accumulations of CO₂. It measures the extent to which the marginal increase in the atmospheric accumulations of CO₂ at time t induces the marginal decrease in the welfare level of country v , including those of all future generations.

Suppose ψ_t^v is the price in units of utility charged to the emission of CO₂ in country v at time t . The resulting pattern of consumption, production, and the level of CO₂ emissions

in country V are obtained as the optimum solution for the following maximum problem:

Find the combination $(c^V, x^V, a_c^V, a_p^V, a^V)$ of consumption vector c^V , production vector x^V , and emissions, a_c^V, a_p^V, a^V , that maximizes the utility of country V

$$u_*^V = \phi^V(V)u^V(c^V, a_c^V) - \psi^V a^V$$

subject to the constraints that

$$pc^V = px^V$$

$$f^V(x^V, a_p^V) \leq K^V$$

$$a^V = a_c^V + a_p^V,$$

where V and ψ^V are given.

The optimum conditions are

$$\alpha^V \phi^V(V)u_{c^V}^V(c^V, a_c^V) \leq p \quad (\text{mod. } c^V)$$

$$\alpha^V \phi^V(V)u_{a_c^V}^V(c^V, a_c^V) \leq \theta^V \quad (\text{mod. } a_c^V)$$

$$p \leq r^V f_{x^V}^V(x^V, a_p^V) \quad (\text{mod. } x^V)$$

$$\theta^V \geq r^V [-f_{a_p^V}^V(x^V, a_p^V)] \quad (\text{mod. } a_p^V)$$

$$f^V(x^V, a_p^V) \leq K^V \quad (\text{mod. } r^V),$$

where α^v is the inverse of marginal utility of income, i. e., $\alpha^v = \frac{1}{\lambda^v}$.

Marginality conditions above imply that net profits

$$px^v - \theta^v a_p^v$$

are maximized over the technological possibility set $(x^v, a_p^v) \in T^v$.

The marginal decrease m_t^v in the welfare level of country v at time t induced by the marginal increase in the atmospheric accumulations V_t of CO₂ at time t is given by

$$m_t^v = -\frac{\partial}{\partial V_t} [\varphi^v(V_t) u^v(c_t^v)] = \tau(V_t) \varphi^v(V_t) u^v(c_t^v),$$

which, in units of market prices, may be expressed as

$$\alpha_t^v m_t^v = \tau(V_t) \alpha_t^v \varphi^v(V_t) u^v(c_t^v) = \tau(V_t) y_t^v,$$

where y_t^v is per capita national income of country v at time t .

Let us denote by π_t^v the imputed price of the atmospheric accumulations of CO₂ in units of market prices for country v at time t ; that is,

$$\pi_t^v = \alpha_t^v \psi_t^v.$$

Then, Then we have from the definition of the imputed price that

$$\pi_t^v = \int_t^\infty \alpha_\tau^v m_\tau^v e^{-(\delta + \mu)(\tau - t)} d\tau = \int_t^\infty \tau(V_\tau) y_\tau^v e^{-(\delta + \mu)(\tau - t)} d\tau.$$

By differentiating both sides of this equation with respect to time t , we obtain the following differential equation:

$$\dot{\pi}_t^v = (\delta + \mu)\pi_t^v - \tau(V_t)y_t^v,$$

which is nothing but the Euler-Lagrange differential equation in the calculus of variations or the Ramsey-Keynes equation in the theory of optimum economic growth.

To clarify the intrinsic meaning of the imputed price of the atmospheric accumulations of CO₂, we introduce a virtual capital market at time t that is perfectly competitive and the atmospheric accumulations of CO₂ are transacted as an asset. The imputed price π_t^v at time t is identified with the market price. Consider the situation in which the unit of such an asset is held for a short time period $[t, t + \Delta t]$, ($\Delta t > 0$). The gains obtained by holding such an “asset” are composed of “capital gains” $\Delta\pi_t^v = \pi_{t+\Delta t}^v - \pi_t^v$ and “earnings” $\tau(V_t)y_t^v\Delta t$, minus “depreciation charges” $\mu\pi_t^v\Delta t$; that is,

$$\Delta\pi_t^v + \tau(V_t)y_t^v\Delta t - \mu\pi_t^v\Delta t.$$

On other hand, the cost of holding such an “asset” for the time period $[t, t + \Delta t]$ is “interest” payment $\delta\pi_t^v\Delta t$, where the social rate of discount is identified with the “market rate of interest”. Hence, on the virtual capital market, these two amounts become equal; that is,

$$\Delta\pi_t^v + \tau(V_t)y_t^v\Delta t - \mu\pi_t^v\Delta t = \delta\pi_t^v\Delta t,$$

which, by dividing both sides by Δt and taking the limit as $\Delta t \rightarrow 0$, yields the above differential equation.

It may be noted that the time derivative in this differential equation refers to the fictitious time of the virtual capital market at time t and remains valid only at time t . For any

future time τ ($\tau > t$), the equation has to be modified by taking into account the changing circumstances due to the accumulation of capital.

We define that the imputed price π_t^v of the atmospheric accumulations of CO₂ for country v is at the *sustainable* level at time t , if $\dot{\pi}_t^v = 0$ at time t . That is,

$$\pi_t^v = \theta_t^v, \quad \theta_t^v = \frac{\tau(V_t)}{\delta + \mu} y_t^v \text{ at time } t.$$

A time-path (V_t) of the atmospheric accumulations of CO₂ is defined sustainable for country v , if the imputed price π_t^v for country v is at the sustainable level at all times t . It is defined *sustainable*, if it is sustainable for all countries in the world.

The discussion above may be summarized as the following proposition:

Proposition 1. *Sustainable time-paths (V_t) of the atmospheric accumulations of CO₂ are obtained as the competitive market equilibrium under the system of proportional carbon taxes, where, in each country v , the carbon taxes are levied at the rate θ^v that is proportional to the per capita national income y^v of each country v , with the discounted present value $\frac{\tau(V)}{\delta + \mu}$ of the impact coefficient of global warming $\tau(V)$ as the coefficient of proportion; that is,*

$$\theta^v = \frac{\tau(V)}{\delta + \mu} y^v,$$

where $\tau(V)$ is the impact coefficient of global warming, δ is the social rate of discount, and μ is the rate at which atmospheric is annually absorbed by the oceans.

Global Warming and Forests

The economic analysis of global warming, as developed in the previous sections, may be extended to examine the role of terrestrial forests in moderating processes of global warming, on the one hand, and in affecting the level of the welfare of people in the society by providing a decent and cultural environment, on the other.

In the simple, dynamic analysis of global warming introduced in the previous sections, we have assumed that the combustion of fossil fuels is the only cause for atmospheric instability and that the surface ocean is the only reservoir of carbon on the earth's surface that exchanges carbon with the atmosphere. In this section, we consider the role of terrestrial forests, particularly tropical rain forests, in stabilizing the processes of atmospheric equilibrium.

Terrestrial forests are regarded as social common capital and managed by social institutions with an organizational structure similar to that of private enterprise except for the manner in which prices of the forests themselves and products from the forests are determined. We assume that the amount of atmospheric absorbed by the terrestrial forest per hectare in each country v is a certain constant on the average to be denoted by γ^v ($\gamma^v \wedge 0$).

Then the basic dynamic equation concerning the change in the atmospheric concentrations of may be modified to take into account the amount of atmospheric CO₂ absorbed by terrestrial

forests. We have

$$\dot{V}_t = a_t - \sum_v \gamma_t^v R_t^v - \mu V_t,$$

where a_t is total CO₂ emissions in the world;

$$a_t = \sum_v a_t^v,$$

R_t^v is the acreages of terrestrial forests of country v , μ is the rate at which atmospheric CO₂ is absorbed by the oceans. We assume that the carbon sequester rate γ^v for temperate forests is around 7.5 tC/ha/yr and for tropical rain forests, it is assumed at 9.6-10.0 tC/ha/yr.

The change in the acreages of the terrestrial forests R^v in each country v is determined first by the levels of reforestation activities and secondly by various economic activities carried out in country v during the year in question—particularly by agricultural and lumber industries and by processes of urbanization. We denote by z_t^v the acreages of terrestrial forests annually reforested and by b_t^v the acreages of terrestrial forests in country v annually lost due to economic activities. Then the acreages of terrestrial forests R_t^v in each country v are subject to the following differential equations:

$$\dot{R}_t^v = z_t^v - b_t^v.$$

Specifications for Utility Functions

We assume that the utility level of each country v is influenced by the acreages of terrestrial forests R^v in country v , in addition to the atmospheric concentrations of CO₂, V . That is, the

utility function for each country ν is expressed in the following manner:

$$u^\nu = u^\nu(c^\nu, a_c^\nu, R^\nu, V),$$

where c^ν is the vector of goods consumed in country ν , a_c^ν is the amount of CO₂ emitted by the consumers in country ν , R^ν is the acreages of terrestrial forests in country ν , and V is the atmospheric concentrations of CO₂ accumulated in the atmosphere, all at time t .

For each country ν , we assume that utility function $u^\nu(c^\nu, a_c^\nu, R^\nu, V)$ is strongly separable with respect to (c^ν, a_c^ν) , and R^ν , thus :

$$u^\nu(c^\nu, a_c^\nu, R^\nu, V) = \phi^\nu(V)\varphi^\nu(R^\nu)u^\nu(c^\nu, a_c^\nu).$$

As with the case discussed in the previous sections, the function $\phi^\nu(V)$ expresses the extent to which people in country ν are adversely affected by global warming, which is referred to as the impact index of global warming. Similarly, the function $\varphi^\nu(R^\nu)$ expresses the extent to which people in country ν are positively affected by the presence of the terrestrial forests in country ν , which is referred to as the impact index of forests. We assume that the impact indices, $\phi^\nu(V)$ and $\varphi^\nu(R^\nu)$, satisfy the following conditions:

$$\phi^\nu(V) > 0, \phi^{\nu'}(V) < 0, \phi^{\nu''}(V) < 0; \varphi^\nu(R^\nu) > 0, \varphi^{\nu'}(R^\nu) > 0, \varphi^{\nu''}(R^\nu) < 0.$$

The impact coefficients of global warming and forests are, respectively, defined by

$$\tau^\nu(V) = -\frac{\phi^{\nu'}(V)}{\phi^\nu(V)}, \quad \tau^\nu(R^\nu) = \frac{\varphi^{\nu'}(R^\nu)}{\varphi^\nu(R^\nu)}.$$

We assume that the impact coefficients of global warming $\tau^\nu(V)$ are identical for all countries ν :

$$\tau^v(V) = \tau(V) \text{ for all } V > 0.$$

The impact coefficient functions, $\tau(V)$ and $\tau^v(R^v)$, satisfy the following conditions:

$$\tau(V) > 0, \tau'(V) > 0; \tau^v(R^v) > 0, \tau^{v'}(R^v) > 0.$$

We assume that, for each country v , the utility function $u^v(c^v, a_c^v)$ satisfies the conditions (U1)–(U4), as introduced above.

The Consumer Optimum

The consumers in country v would choose the vector of consumption c^v and CO₂ emissions a_c^v that maximizes country v 's utility function

$$u^v(c^v, a_c^v, R^v, V) = \phi^v(V)\phi^v(R^v)u^v(c^v, a_c^v)$$

subject to the budget constraints

$$pc^v + \theta^v a_c^v = y^v,$$

where y^v is national income of country v in units of world prices.

The linear homogeneity hypothesis for the utility function $u^v(c^v, a_c^v)$ implies that

$$\alpha^v \phi^v(V)\phi^v(R^v)u^v(c^v, a_c^v) = pc^v + \theta^v a_c^v = y^v,$$

where α^v is the inverse of the marginal utility of income y^v of country v .

Specifications for Production Possibility Sets

The conditions concerning the production of goods in each country v are specified by the

production possibility set A^v in exactly the same manner as in the previous sections.

In each country v , the minimum quantities of factors of production needed to produce goods by the vector of production with the use of the natural resources of the forests by the amount b^v and the CO₂ emission at the level a_p^v are specified by a vector-valued function

$$f^v(x^v, b^v, a_p^v) = (f_f^v(x^v, b^v, a_p^v)).$$

Similarly, the minimum quantities of factors of production needed to engage in reforestation activities at the level z^v are specified by a vector-valued function $g^v(z^v)$.

We assume that marginal rates of substitution between the production of goods, the use of the natural resources of forests, reforestation activities, and the emission of CO₂ are smooth and diminishing, trade-offs always exist between them, and the conditions of constant returns to scale prevail. That is, we assume that

(T1) $f^v(x^v, b^v, a_p^v)$ and $g^v(z^v)$ are defined, positive-valued, continuous, and continuously twice-differentiable for all $(x^v, b^v, a_p^v) \geq 0$ and $z^v \geq 0$, respectively.

(T2) $f_{x^v}^v(x^v, b^v, a_p^v) \geq 0$, $f_{b^v}^v(x^v, b^v, a_p^v) < 0$, $f_{a_p^v}^v(x^v, b^v, a_p^v) < 0$ for all $(x^v, b^v, a_p^v) \geq 0$

$$g_{z^v}^v(z^v) \geq 0 \text{ for all } z^v \geq 0.$$

(T3) $f^v(x^v, b^v, a_p^v)$ and $g^v(z^v)$ are strictly quasi-convex with respect to (x^v, b^v, a_p^v) and z^v , respectively.

(T'4) $f^v(x^v, b^v, a_p^v)$ and $g^v(z^v)$ are homogeneous of order 1 with respect to (x^v, b^v, a_p^v) and z^v , respectively.

The production possibility set A^v is given by

$$A^v = \left\{ (x^v, z^v, b^v, a_p^v) : (x^v, z^v, b^v, a_p^v) \geq 0, f^v(x^v, b^v, a_p^v) + g^v(z^v) \leq K^v \right\},$$

where K^v is the vector of endowments of fixed factors of production in country v .

Postulates (T'1-T'3), as specified above, imply that the production possibility set A^v is a closed convex set of (x^v, z^v, b^v, a_p^v) .

The Producer Optimum

Suppose that prices of goods are given by p and the imputed price of forests in each country by π^v , whereas carbon taxes at the rate θ^v are levied on the emission of CO_2 in each country v .

Forests are regarded as social common capital, and there are no markets on which either the ownership of forests or the entitlements for the products from forests are transacted. Hence, prices of forests are generally not market prices, but rather imputed prices. The imputed price of the ownership of a particular forest is the discounted present value of the stream of the marginal utilities of the forest and the expected value of the entitlements for the natural resources in forest in the future.

The producers in country v would choose those combinations (x^v, z^v, b^v, a_p^v) of vectors of production x^v , levels of reforestation z^v , use of resources of forests b^v , and CO₂ emissions a_p^v that maximize net profits

$$px^v + \pi^v(z^v - b^v) - \theta^v a^v$$

over $(x^v, z^v, b^v, a_p^v) \in T^v$.

where r^v is the vector of imputed rental prices of factors of production.

Imputed Prices of Capital in General

The imputed price, in units of the utility, of each kind of capital at time t , ψ_t , is the discounted present value of the marginal increases in total utility in the future due to the marginal increase in the stock of that kind of capital at time t . When we denote by r_τ the marginal increase in the total utility at future time τ , the imputed price at time t , ψ_t , is given by

$$\psi_t = \int_t^\infty r_\tau e^{-(\delta + \mu)(\tau - t)} d\tau.$$

By differentiating both sides of this equation, we obtain the following differential equation:

$$\dot{\psi}_t = (\delta + \mu)\psi_t - r_t.$$

We suppose that capital is transacted as an asset on a virtual capital market that is perfectly competitive and the imputed price ψ_t is identified with the market price at time t .

Consider the situation in which the unit of such an asset is held for the short time period $[t, t + \Delta t]$ ($\Delta t > 0$). The gains obtained by holding such an asset are composed of "capital gains" $\Delta\psi_t = \psi_{t+\Delta t} - \psi_t$ and "earnings" $r_t \Delta t$; that is,

$$\Delta\psi_t + r_t \Delta t.$$

On other hand, the costs of holding such an asset for the time period $[t, t + \Delta t]$ consist of "interest payments" $\delta\psi_t \Delta t$ and "depreciation charges" $\mu\psi_t \Delta t$, where the social rate of discount δ is identified with the market rate of interest; that is,

$$\delta\psi_t \Delta t + \mu\psi_t \Delta t.$$

On the virtual capital market, these two amounts become equal; that is,

$$\Delta\psi_t + r_t \Delta t = \delta\psi_t \Delta t + \mu\psi_t \Delta t.$$

By dividing both sides of this equation by Δt and taking the limit as $\Delta t \rightarrow 0$, we obtain the above relation.

We define that the imputed price ψ_t is at the sustainable level at time t , if it remains stationary at time t ; i. e.,

$$\dot{\psi}_t = 0 \quad \text{at time } t,$$

where it may be reminded that $\dot{\psi}_t$ refers to the time derivative with respect to the time of the virtual capital market at time t .

The imputed price ψ_t is at the sustainable level at time t , if, and only if,

$$\psi_t = \frac{r_t}{\delta + \mu} \text{ at time } t,$$

where r_t is the marginal increase in total utility due to the marginal increase in the stock of capital of that kind at time t .

Imputed Prices of Atmospheric Concentrations of CO₂ and Forests

Consider the situation in which a combination $(c^v, a_c^v, x^v, z^v, b^v, a_p^v)$ of vectors of consumption and production, c^v, x^v , level of reforestation activities z^v , CO₂ emissions a_c^v , and a_p^v , $a^v = a_c^v + a_p^v$, is chosen in country v . Imputed prices of atmospheric concentrations of CO₂ and forests are defined as follows.

Suppose CO₂ emissions in country v , a^v , are increased by a marginal amount. This would induce a marginal increase in the aggregate amount of CO₂ emissions in the world, causing a marginal increase in the atmospheric level of CO₂. The resulting marginal increase in the degree of future global warming would cause a marginal decrease in country v 's utility.

The marginal decrease in country v 's utility due to the marginal increase in emissions today in country v is given by the partial derivative, with minus sign, of utility of country v

$$u^v = \phi^v(V)\phi^v(R^v)u^v(c^v, a_c^v)$$

with respect to atmospheric accumulations of CO₂, V ; that is,

$$-\frac{\partial u^v}{\partial V} = \tau(V)\phi^v(V)\phi^v(R^v)u^v(c^v, a_c^v),$$

where $\tau(V)$ is the impact coefficient of global warming.

We assume that future utilities of country v are discounted at the social rate of discount δ that is assumed to be positive and identical for all countries in the world. We have assumed that the rate at which atmospheric carbon dioxide is annually absorbed by the oceans is a certain constant μ . Hence, for each country v , the sustainable imputed price ψ^v of the atmospheric accumulations of CO₂, in units of utility of country v , is given by the discounted present value of the marginal decrease in utility of country v due to the marginal increase in emissions in country v today ; that is,

$$\psi^v = \frac{\tau(V)}{\delta + \mu} \phi^v(V) \phi^v(R^v) u^v(c^v, a_c^v).$$

Hence, the sustainable imputed price θ^v of the atmospheric accumulations of for country V , in units of world prices, is given by

$$\theta^v = \alpha^v \psi^v = \frac{\tau(V)}{\delta + \mu} y^v,$$

where α^v is the inverse of the marginal utility of country V .

Similarly, the imputed prices of forests, in units of world prices, are defined as follows. Suppose the acreages of forests of country V , R^v , are increased by a marginal amount. This would induce a marginal increase in the level of the utility of country v , on the one hand, and a marginal increase in the utility of country v in the future due to the marginal decrease in the atmospheric level of induced by the absorbing capacity of forests in country

v , on the other.

The first component is the marginal utility with respect to the acreages of forests of country v , R^v , in units of world prices. It is given by

$$\frac{\partial(\alpha^v u^v)}{\partial R^v} = \tau^v(R^v) \alpha^v \phi^v(V) \phi^v(R^v) u^v(c^v, a_c^v) = \tau^v(R^v) y^v.$$

The second component is the marginal increase in country v 's utility in the future due to the marginal decrease in the atmospheric level of V induced by the absorbing capacity of forests in country v . It is given by

$$\gamma^v \theta^v = \gamma^v \frac{\tau(V)}{\delta + \mu} y^v.$$

Hence, the sustainable imputed price π^v of forests of country v , in units of world prices, is given by the discounted present value of the sum of these two components:

$$\pi^v = \frac{1}{\delta} [\tau^v(R^v) + \gamma^v \frac{\tau(V)}{\delta + \mu}] y^v.$$

The discussion above may be summarized in the following proposition.

Proposition 2. *The optimum conditions for the sustainable time-path of atmospheric concentrations of CO₂ corresponds precisely to the optimum conditions for the competitive market equilibrium under the following system of proportional carbon taxes for the emission of CO₂ and tax-subsidy measures for the reforestation and depletion of resources of forests:*

- (i) *In each country v , the carbon taxes are levied with the rate that is proportional to the per capita national income y^v :*

$$\theta^v = \frac{\tau(V)}{\delta + \mu} y^v,$$

where $\tau(V)$ is the impact coefficient of global warming, δ is the social rate of discount, and μ is the rate at which atmospheric CO_2 is annually absorbed by the oceans.

(ii) In each country v , tax-subsidy arrangements are made for the depletion of resources and reforestation of forests with the rate π^v that is proportional to the national income y^v , to

be given by

$$\pi^v = \frac{1}{\delta} [\tau^v(R^v) + \gamma^v \frac{\tau(V)}{\delta + \mu}] y^v,$$

where $\tau^v(R^v)$, γ^v are, respectively, the impact coefficient and carbon sequester rate for forests in country v .

International Fund for Atmospheric Stabilization

The divergence in economic performance between developed countries and developing countries has steadily widened in the last several decades, and various institutional and policy measures that have been devised internationally or bilaterally have not had much impact in narrowing the gap between these two groups of countries. The introduction of the proportional carbon tax system as envisioned here, in spite of the implicit recognition of the equity aspect in its design, may tend to worsen the relative position of developing countries, at least in the short-run. It would be desirable, therefore, to supplement the carbon tax system with the international redistributive scheme that would have significant impact in narrowing the gap between the stages of economic development of various countries involved.

The International Fund for Atmospheric Stabilization is an institutional framework in which it is possible to combine an international arrangement to stabilize atmospheric equilibrium with a redistributive scheme to help developing countries to accelerate processes of economic development.

The International Fund for Atmospheric Stabilization presupposes that each country adopts the proportional carbon tax system under which emissions of carbon dioxide and other greenhouse gases are charged a levy evaluated at the imputed prices proportional to the per capita level of national income and a charge (or a subsidiary payment) is made for the depletion (or the afforestation) of terrestrial forests, again based upon the evaluation at the imputed prices of terrestrial forests that are proportional to the per capita level of national income, as in detail discussed in the present paper.

The tax revenues from the proportional carbon tax system are principally put into the general revenue account of each government, preferably to be partly earmarked for the purposes of restoring the natural and ecological environments, and for encouraging private economic agents to develop those technological and institutional knowledge that are crucial in restoring equilibrium conditions in the global environment.

Each country then transfers a fixed portion, say 5%, of the net revenue from the carbon tax system to the International Fund for Atmospheric Stabilization . The total amount transferred to the International Fund for Atmospheric Stabilization from individual countries then would be allocated to developing countries according to a certain predetermined schedule, properly taking into account the per capita levels of national income and the size of

population. Developing countries may use the amounts transferred from the International Fund for Atmospheric Stabilization for the purposes which they think appropriate, preferably for compensating those who would suffer from the phenomena of global environmental disequilibrium and incur the hardships by the implementation of the carbon tax system, for restructuring industrial organizations and social infrastructure, and for introducing substitutional energy sources and energy-saving technologies.

It is difficult to imagine that the International Fund for Atmospheric Stabilization or similar international arrangements on the global scale may be instituted in any immediate future. Whether such international arrangements may be effectively implemented or not depends to a significant extent upon the degree of awareness on the part of the general public concerning the enormous burden and costs future generations will have to suffer from the phenomena of global warming and other global environmental disequilibrium.

The strenuous effort by a large number of geo-scientists, ecologists, and other scientists to clarify the mechanism of global warming and to identify the specific implications of global warming and other environmental issues for ecological, biological, social, and cultural life on Earth has had a significant impact to the awareness and consciousness of the general public and the national governments. The numerous conferences and symposia organized by various international organizations, such as the 1991 Rio Conference and the Intergovernmental Panel on Climate Change, particularly the Kyoto Protocol of 1997, have substantially altered the perception of the international community as regards the plausibility and danger of global warming and other atmospheric disequilibria.

All these help the national governments involved to search for those policy and institutional arrangements that will make the practical implementation of the International Fund for Atmospheric Stabilization or similar international agreements feasible from economic, social, and political points of view. It would not be too optimistic to expect to have the International Fund for Atmospheric Stabilization or a similar framework to be instituted within a foreseeable period, though not in the immediate future.

A Hypothetical Case

A hypothetical case of the incidences of the proportional carbon taxes under the system of proportional carbon taxes for the emission of CO₂ and tax-subsidy measures for the reforestation and depletion of resources of forests are presented, all in terms of the statistical data of 2005 (in US\$).

Table 1
Incidences of proportional carbon taxes with the coefficient of proportion 0.01
including all radiative forcing agents (RFA)

Country	National Income per capita	RFA Increase per capita(Ct)	Imputed Price per Ct	Carbon Taxes per capita
United States	42,000	5.90	420	2,500
Canada	34,000	6.20	340	2,100
United Kingdom	32,000	3.00	320	950
France	31,000	2.20	310	680
Germany	31,000	3.20	310	980
Italy	28,000	2.20	280	600
Netherlands	35,000	3.60	350	1,200
Sweden	32,000	1.90	320	610
Norway	48,000	1.60	480	760
Finland	31,000	2.00	310	610
Denmark	34,000	3.20	340	1,100
Indonesia	3,100	1.70	30	50
Japan	31,000	2.70	310	840
Korea	21,000	2.60	210	560
Malaysia	11,000	1.90	110	210
Philippines	3,200	0.30	30	8
Singapore	40,000	3.20	400	1,300
Thailand	6,900	1.20	70	80
India	2,200	0.30	20	7
China	4,100	1.10	40	40
Australia	33,000	7.10	330	2,300
New Zealand	23,000	3.50	230	790

Sources) UNFCCC, World Development Indicators, etc.

Table 2
Incidences of Tax-Subsidy Measures for the Reforestation
and Depletion of Resources of Forests

Country	Forest and	Net Annual	Imputed	Assessment	
	Woodlands	Reforestation	Price	Total	Per Capita
	[Million ha]	[1000 ha]	[Per ha]	[Million Dollars]	[Dollars]
United States	303	159	42,000	6,627	22
Canada	310	0	34,000	0	0
United Kingdom	3	10	32,000	321	5
France	16	41	31,000	1,264	21
Germany	11	0	31,000	0	0
Italy	10	106	28,000	2,929	50
Netherlands	0	1	35,000	35	2
Sweden	28	11	32,000	351	39
Norway	9	17	48,000	808	175
Finland	23	5	31,000	153	29
Denmark	1	3	34,000	102	19
Indonesia	88	-1,871	9,300	-17,120	-77
Japan	25	-2	31,000	-62	0
Korea	6	-7	21,000	-149	-3
Malaysia	21	-140	33,000	-4,679	-179
Philippines	7	-157	9,600	-1,507	-18
Singapore	0	0	40,000	0	0
Thailand	15	-59	20,700	-1,220	-19
India	68	29	2,200	64	0
China	197	4,058	4,100	16,678	13
Australia	164	-193	33,000	-6,319	-310
New Zealand	8	17	23,000	388	94

Source) World Resources Institute, etc.

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